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TOTAL ROMAN DOMINATION ON KNESER GRAPHS

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Abstract. A Roman dominating function (RDF) on a graph G is a function $f : V(G) \longrightarrow \{0, 1, 2\}$ such that any vertex v with $f(v) = 0$ is adjacent to at least one vertex w with $f(w) = 2$. In addition, if the subgraph of G induced by the set of all vertices for which $f \neq 0$ has no isolated vertices then f is called a total Roman dominating function (TRDF). If f is an RDF (TRDF) then the least value of $\sum_{u\in V} f(u)$ is called Roman domination number (total Roman domination number) of G and is denoted by $\gamma_R(G)$ ($\gamma_{tR}(G)$). Let $G = G(n, k, 0)$ be a Kneser graph, where n, k are positive integers. In this paper we present some bounds for $\gamma_{tR}(G(n, k, 0))$ for $k^2 < n < k^2 + k$. In particular we show that $\gamma_{tR}(G(k^2 + k - 1, k, 0)) = 2(k + 2)$ and for $n \geqslant 2k + 1$, $\gamma_R(G(n, k, 0)) \geq max\{\gamma_R(G(n - 1, k - 1, 0)), \gamma_R(G(n, k - 1, 0))\}$.

1. Introduction

Let $G = (V, E)$ be a graph with vertex set V and edge set E. The cardinality of V is called the order of G. Given positive integers k and n, a Kneser graph denoted by $G(n, k, 0)$, is defined as a graph where the vertices are all k-subsets of $[n] = \{1, 2, \ldots, n\}$ and two vertices are adjacent if the corresponding k-subsets are disjoint, hence $G(n, k, 0)$ is a $\binom{n-k}{k}$-regular graph of order $\binom{n}{k}$. Clearly $G(n, 1, 0)$ is the complete graph on n vertices and $G(2n + 1, n, 0)$ is the odd graph, denoted by O_n, in particular $O_2 = G(5, 2, 0)$ is the Petersen graph (see figure 1). Let G and H be two graphs. A homomorphism from G to H is defined as a function $f : V(G) \longrightarrow V(H)$ such that any two adjacent

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vertices in G maps to two adjacent vertices in H. A subset S of vertices of a graph G is called a dominating set of G, if every vertex of G either is in S or is adjacent to at least one of the vertices of S. The domination number of G is the minimum cardinality of all dominating sets of G and is denoted by $\gamma(G)$. For the necessary concepts and terminologies we rely on [11].

A Roman dominating function (RDF) on a graph $G = (V, E)$ is a function $f : V \longrightarrow \{0, 1, 2\}$ having the property that any vertex v with $f(v) = 0$ is adjacent to at least one vertex w with $f(w) = 2$. The weight of a RDF is the value $f(V) = \sum_{u \in V} f(u)$. The minimum weight of all Roman dominating functions on G is called the Roman domination number (RDN) of G and is denoted by $\gamma_R(G)$ [4]. Any graph G with $\gamma_R(G) = 2\gamma(G)$ is called a Roman graph. A total Roman dominating function ($TRDF$) on G is an RDF on G such that the subgraph of G induced by the set of vertices with nonzero RDF value, has no isolated vertex [1]. The weight of a $TRDF$ is defined as the weight of the RDF, given above. The total Roman domination number of G, denoted by $\gamma_{tR}(G)$ is the minimum weight of all total Roman dominating function on G.

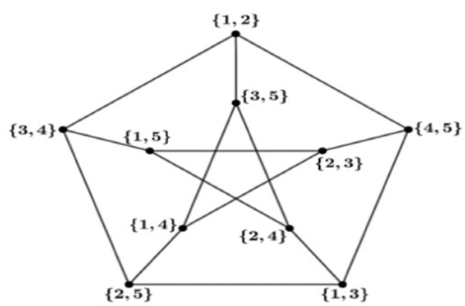

FIGURE 1. Kneser graph $G(5, 2, 0)$

The Roman domination number of a graph introduced by Cockayne et al. [4] and the total Roman domination number of a graph introduced by Liu and Chang [6]. The relation between $RDN(TRDN)$ and domination number (total domination number) of a graph is studied by Ahangar et al. [1]. Tatjana et al. investigated $TRDN$ on Kneser graphs and showed that $\gamma_R(G(n, k, 0)) = \gamma_{tR}(G(n, k, 0)) = 2(k + 1)$ for $n \geqslant k^2 + k$ [10]. In this paper we show that $\gamma_{tR}(G(k^2 + k - 1, k, 0)) = 2(k + 2)$. We also provide some bounds for $\gamma_{tR}(G(n, k, 0))$, where $k^2 < n < k^2 + k$. In particular, we show that for $n \geqslant 2k+1$, $\gamma_R(G(n, k, 0)) \geq max\{\gamma_R(G(n-1, k-1, 0)), \gamma_R(G(n, k-1, 0))\}$. See [2, 3, 4, 8, 9] for more details about Roman and total Roman domination numbers. First note the following two theorems that we need.

Theorem 1.1. *For any connected graph G, we have $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$* [4].

Theorem 1.2. *If G is a graph without isolated vertices, then $\gamma_R(G) \leq \gamma_{tR}(G)$* [4].

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2. Main Results

To give our results first we remind the concept of *upward shadow* of a $(k-1)$-subset of $[n]$ as the set of all k-subsets of $[n]$ that contain it [7]. Clearly the upward shadow of a fixed $(k-1)$-subset is of size $n-k+1$.

Lemma 2.1. *Let $n \geqslant 2k+1$ and f be an RDF on $G(n,k,0)$. If the set $\{v \in G(n,k,0); f(v) \neq 0\}$ does not contain upward shadow of any $(k-1)$-subset of $[n]$, then f can be extended to an RDF for $G(n+1,k,0)$.*

Proof. The graph $G(n,k,0)$ is a subgraph of $G(n+1,k,0)$. We define the function $g : V(G(n+1,k,0)) \longrightarrow \{0,1,2\}$ as $g(v) = f(v)$ for each $v \in V(G(n,k,0))$ and $g(v) = 0$ otherwise. We show that g is an RDF on $G(n+1,k,0)$. So we should show that any vertex $v \in V(G(n+1,k,0))$ with $g(v) = 0$ is adjacent to a vertex, say w, with $g(w) = 2$. It is sufficient to prove the claim is true for any vertices not in $V(G(n,k,0))$. Let $v = \{v_1,\ldots,v_{k-1},n+1\}$, where $v_1,\ldots,v_{k-1} \in [n]$ be a vertex of $V(G(n+1,k,0))$ not belong to the subgraph $G(n,k,0)$ with $g(v) = 0$. As is supposed the set $\{v_1,\ldots,v_{k-1}\}$ has no upward shadow in $V(G(n,k,0))$ with nonzero f. So there is a vertex in the upward shadow of $\{v_1,\ldots,v_{k-1}\}$, say $v^* = \{v_1,\ldots,v_{k-1},v_j\}$ with $f(v^*) = 0$. Since f is an RDF on $G(n,k,0)$, the vertex v^* has an adjacent vertex, say $w \in G(n,k,0)$, where $f(w) = 2$. Hence w and v^* have no intersection as k-sets. This, as well, shows that v and w have no intersection as k-sets. This completes the prove. □

Theorem 2.2. *For $G = G(n,k,0)$ and $n = k^2 + i\,(1 \leq i < k)$, we have*
i. If $\lceil k/2\rceil \leq i < k$, then $\gamma_{tR}(G) \leq 2(k+2)$.
ii. If $1 \leq i < \lceil k/2\rceil$, then $\gamma_{tR}(G) \leq 4k$.

Proof. Let $n = k^2 + \lceil \frac{k}{2} \rceil$ and consider

$$C_i = \{a_{(i-1)k+1}, a_{(i-1)k+2}, \ldots, a_{ik}\}; 1 \leq i \leq k,$$

$$C_{k+1} = \{a_{k^2+1}, \ldots, a_{k^2+\lceil \frac{k}{2} \rceil}, a_1, \ldots, a_{k-\lceil \frac{k}{2} \rceil}\},$$

$$C_{k+2} = \{a_{k^2+1}, \ldots, a_{k^2+\lceil \frac{k}{2} \rceil}, a_{k-\lceil \frac{k}{2} \rceil+1}, \ldots, a_{2k-2\lceil \frac{k}{2} \rceil}\}.$$

1. We show that the set $S = \{C_1, C_2, \ldots, C_k, C_{k+1}, C_{k+2}\}$ is a dominating set for G. Let x be a vertex of G. Either $x \cap C_i \neq \emptyset$ for all $1 \leq i \leq k$, in which x is dominated by C_{k+1} or C_{k+2}, or there exist a positive integer $j\,(1 \leq j \leq k)$ such that $x \cap C_j = \emptyset$, in which x is dominated by C_j. Consider the function $f : V(G) \longrightarrow \{0,1,2\}$, defined by $f(v) = 2$ if $v \in S$ and $f(v) = 0$ otherwise. Since the subgraph of G induced by S is connected, the defined function f is a $TRDF$ and we have

$$\gamma_{tR}(G(n,k,0)) \leq 2 \mid S \mid = 2(k+2).$$

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The set S does not contain upward shadow of any $(k-1)$-subset of $[n]$, hence by lemma 2.1 for $n = k^2 + i\,(\lceil \frac{k}{2} \rceil \leq i < k)$ we have $\gamma_{tR}(G(n,k,0)) \leq 2(k+2)$.

2. Let $n = k^2+1$. Consider $S = \{C_1, C_2, \ldots, C_k\}$ and

$$S' = \{\{a_{k^2+1}, y_1, \ldots, y_{k-1}\};\, y_i \in C_1, 1 \leq i \leq k-1\}.$$

We show that $S \cup S'$ is a dominating set for G. Let x be a vertex of G. If there exist an index $j\,(1 \leq j \leq k)$ such that $x \cap C_j = \emptyset$, then x is dominated by C_j, otherwise $|x \cap C_i| = 1$ for all $1 \leq i \leq k$. Let $x \cap C_1 = \{a_l\}$, where $1 \leq l \leq k$, then x is dominated by $\{a_{k^2+1}, a_1, \ldots, a_{l-1}, a_{l+1}, \ldots, a_k\}$. Consider the function $f : V(G) \longrightarrow \{0,1,2\}$ defined by $f(v) = 2$ if $v \in S \cup S'$ and $f(v) = 0$ otherwise. Since the subgraph of G induced by the set $S \cup S'$ is connected, the defined function f is a $TRDF$ and we have

$$\gamma_{tR}(G(k^2+1,k,0)) \leq 2(\mid S' \cup S \mid) = 2(k+k) = 4k.$$

Since $S \cup S'$ does not contain upward shadow of any $(k-1)$-subset of $[n]$, by lemma 2.1 for $1 \leq i < \lceil \frac{k}{2} \rceil$, we have $\gamma_{tR}(G(n,k,0)) \leq 4k$. $\square$

Theorem 2.3. *If $n \geq \frac{3k^2}{4} + k$, $k = 2m$ and $m \geq 2$, then $\gamma_{tR}(G) \leq 3k+2$.*

Proof. Let $n = \frac{3k^2}{4} + k$ and $S = \{C_1, C_2, \ldots, C_{3m+2}\}$, where C_i's are m-sets $(1 \leq i \leq 3m+2)$ and $C_i \cap C_j = \emptyset$ for $i \neq j$. Consider $L_i = \{C_{3i-2}, C_{3i-1}, C_{3i}\}$ for $1 \leq i \leq m$. We show that $S' = (\bigcup_{i=1}^{m} P_2(L_i)) \cup \{C_{3m+1} \cup C_{3m+2}\}$ is a dominating set for G, where $P_2(L_i)$ is the set of all 2-subsets of L_i. Let x be a vertex of G. If $|x \cap L_i| \geq 2$ for all i, then $x \cap \{C_{3m+1} \cup C_{3m+2}\} = \emptyset$ and x is dominated by $\{C_{3m+1} \cup C_{3m+2}\}$. Otherwise, there exists an integer j $(1 \leq j \leq m)$, such that $|x \cap L_j| < 2$ and x is dominated by at least one element of $P_2(L_j)$. Consider the function $f : V(G) \longrightarrow \{0,1,2\}$ defined by $f(v) = 2$ if $v \in S'$ and $f(v) = 0$ otherwise. Since the subgraph of G induced by the set S' is connected, the defined function f is a $TRDF$ and we have

$$\gamma_{tR}(G(\frac{3k^2}{4}+k),k,0) \leq 2 \mid S' \mid = 2(3m+1) = 2(3(\frac{k}{2})+1) = 3k+2.$$

Since S does not contain upward shadow of any $(k-1)$-subset of $[n]$, by lemma 2.1 we have

$$\gamma_{tR}(G(n,k,0)) \leq 3k+2\,(n \geq \frac{3k^2}{4}+k).$$

$\square$

Inspired from a theorem given in [10] we give the following theorem:

Theorem 2.4. *If $G = G(k^2+k-1,k,0)$, where $k \geq 4$, then $\gamma_{tR}(G) = 2(k+2)$.*

Proof. By theorem 2.2, we have $\gamma_{tR}(G) \leq 2(k+2)$. Given f be a $TRDF$ on G we have the partition $V(G) = V_0 \cup V_1 \cup V_2$, where $V_i = \{v \in V(G); f(v) = i\}$ for $i = 0,1,2$. In the following we study all possible values of $|V_2|$ in different cases and show that in all cases $f(V) \geq 2(k+2)$. This implies that $\gamma_{tR}(G) \geq 2(k+2)$ and the proof completed.

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1. $|V_2| = 0$. Since f is a $TRDF$ and $V_2 = \emptyset$, we have $V_0 = \emptyset$ and $f(V) = |V_1|$. So

$$|V_1| = |V| = \binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!} \geq \frac{(k^2+k-1)((k^2+k-2)\cdots(k^2))}{k!} > \frac{k^{2k}}{k!}.$$

Since $k \geq 4$, we have $\frac{k^{2k}}{k!} = \frac{k^k}{k!} \times k^k \geq k^k \geq 2(k+2)$. Consequently

$$f(V) =| V_1 |\geq 2(k+2).$$

2. $1 \leq |V_2| \leq 2$. At most $k|V_2|$ different elements of $[n]$ are used in k-subsets of the set V_2. So at least $n - k|V_2| \geq k^2 + k - 1 - (2k) = k^2 - k - 1$ different elements of $[n]$ appear in no vertex of V_2. Let T be the set of these elements. Consider $M = \{\{a_1, \dots, a_{k-1}, t\}; a_i \in [n] \setminus T, t \in T\}$, such that each vertex of M intersects with each vertex of V_2. So no vertices of M is connected to vertices of V_2 hence $M \subseteq V_1$ and we have $|V_1| \geq |M| = k^2 - k - 1$. Therefore

$$f(V) = 2|V_2| + |V_1| \geq 2 + (k^2 - k - 1) = k^2 - k + 1 \geq 2(k+2).$$

3. $3 \leq |V_2| \leq k - 1$. By a similar argument as case 2 there exist at least $2k - 1$ element not in V_2. Consider M as defined above then $|V_1| \geq |M| \geq 2k - 1$, hence

$$f(V) = 2|V_2| + |V_1| \geq 6 + (2k - 1) = 2k + 5 \geq 2(k+2).$$

4. $|V_2| = k$. If the subgraph induced by V_2 be a clique, then $|V_1| = k^k \geq 2k$ and

$$f(V) = 2|V_2| + |V_1| \geq 2k + 2k = 4k \geq 2(k+2).$$

Otherwise there exist at least two vertices u and v in V_2 with non-empty intersection. Let $s \in u \cap v$. Let T be the set as defined in case 2 and consider $M = \{\{s, a_2, \dots, a_{k-1}, t\}; a_i \in [n], t \in T\}$, such that each vertex of M intersects with each vertex of V_2. Clearly there exist at least k elements in V_1 and

$$f(V) = 2|V_2| + |V_1| \geq 2k + k \geq 2(k+2).$$

5. $|V_2| = k + 1$. Let $V_2 = \{C_1, C_2, \dots, C_k, C_{k+1}\}$. Since $|C_i| = k \, (1 \leq i \leq k+1)$ and $n = k^2 + k - 1$ at least two elements of V_2 intersect of size one. Let $C_1 \cap C_2 = \{r\}$, where $r \in [n]$. We show that $|V_1| \geq 2$, which implies $f(V) = 2|V_2| + |V_1| \geq 2(k+1) + 2 = 2k + 4 \geq 2(k+2)$. This ends the proof. We construct two vertices X and Y in V_1 as follows. Let $X = \{x_1, \dots, x_k\}, Y = \{y_1, \dots, y_k\}$ and consider

$$C_i = \{a_{(i-1)k+1}, a_{(i-1)k+2}, \dots, a_{ik}\}; 1 \leq i \leq k+1,$$

Let $x_1 = y_1 = r$. The vertex C_3 has at least $k - 1$ elements, such as a_{2k+1} which are different with r. Let $x_2 = y_2 = a_{2k+1}$. With a similar method we can choose $x_j = y_j$ in $C_{j+1} \, (3 \leq j \leq k-1)$. As is clear C_k and C_{k+1} each has at least two elements not used above. Let $a, b \in C_k$ and $c, d \in C_{k+1}$ be these elements. If for example $a \neq c$, then let $x_{k-1} = a, y_{k-1} = c, x_k = d, y_k = b$. Otherwise we assume that $a = c, b = d$. Since C_k and C_{k+1} are different vertices there exist at least one element say $h \in C_{k+1} \setminus C_k$. If h is not used before, then let $x_{k-1} = a, y_{k-1} = a, x_k = b, y_k = h$. If h is

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used before, then there exists an element say $g \in C_k$, which is not used before, so we may consider $x_{k-1} = a, y_{k-1} = a, x_k = g, y_k = b$.

6. $|V_2| \geq k + 2$. Clearly $f(V) = 2|V_2| + |V_1| \geq 2(k + 2)$. □

Lemma 2.5. *If there exists a surjective homomorphism from a graph G to a graph H, then $\gamma_R(G) \geq \gamma_R(H)$.*

Proof. Suppose that f is an RDF on G and $\phi : V(G) \longrightarrow V(H)$ is a surjective homomorphism. We define a function $g : V(H) \longrightarrow \{0, 1, 2\}$ as follows and show that g is an RDF on H. For any $z \in V(H)$ consider $\phi^{-1}(z) = \{x_{i_1}, \ldots, x_{i_r}\}$ and let $g(z) = max\{f(x_{i_j}); 1 \leq j \leq r\}$. Let $y' \in V(H)$ and $g(y') = 0$. There exists $y \in V(G)$ with $\phi(y) = y'$ and $f(y) = 0$. Since f is an RDF on G there exists a vertex $z \in V(G)$ such that $yz \in E(G)$ and $f(z) = 2$. If we let $\phi(z) = z'$, then $g(z') = 2$ and since ϕ is a homomorphism $y'z'$ is an edge in H which implies that g is an RDF. If f be an RDF with minimum weight then: $\gamma_R(G) = f(V(G)) \geq g(V(H)) \geq \gamma_R(H)$. □

Theorem 2.6. *For $n \geq 2k + 1$, we have*

i $\gamma_R(G(n, k, 0)) \geq \gamma_R(G(n - 1, k - 1, 0))$,

ii $\gamma_R(G(n, k, 0)) \geq \gamma_R(G(n, k - 1, 0))$.

Proof. *i*. The function $\phi : V(G(n, k, 0)) \longrightarrow V(G(n-1, k-1, 0))$ defined as follows is a homomorphism. For any $\{a_{i_1}, a_{i_2}, \ldots, a_{i_k}\} \in V(G(n, k, 0))$ if $a_{i_j} = max\{a_{i_1}, a_{i_2}, \ldots, a_{i_k}\}$, then $\phi(\{a_{i_1}, a_{i_2}, \ldots, a_{i_k}\}) = \{a_{i_1}, \ldots \widehat{a_{i_j}}, \ldots, a_{i_k}\}$, where the hat notation denotes the deleted item ([5, p.148]). Clearly ϕ is surjective. So by above lemma we have

$$\gamma_R(G(n, k, 0)) \geq \gamma_R(G(n - 1, k - 1, 0)).$$

ii. If in above definition of ϕ, we delete a fixed j-th position ($i_1 \leq j \leq i_k$) on each vertex $\{a_{i_1}, a_{i_2}, \ldots, a_{i_k}\} \in V(G(n, k, 0))$, then ϕ is a surjective homomorphism and by above lemma we have $\gamma_R(G(n, k, 0)) \geq \gamma_R(G(n, k - 1, 0))$. □

Theorem 2.7. *If $G = O_k$, then $\gamma_R(G) \leq 2\binom{2k-1}{k-1} + \binom{2k-1}{k-2}$.*

Proof. Consider $[n] = \{a_1, \ldots, a_k, b_1, \ldots, b_k, c\}$ and let

$$V_2 = \{\{a_1, x_1, \ldots, x_{k-1}\} \mid \{x_1, \ldots, x_{k-1}\} \subseteq [n] - \{a_1, c\}\},$$

$$V_1 = \{\{c, a_1, x_1, \ldots, x_{k-2}\} \mid \{x_1, \ldots, x_{k-2}\} \subseteq [n] - \{a_1, c\}\},$$

$$V_0 = V(G) - (V_1 \cup V_2).$$

Since any vertex of V_0 is connected to a vertex of V_2 the function f is an RDF on G and we have:

$$\gamma_R(G) \leq f(V) = 2 \mid V_2 \mid + \mid V_1 \mid = 2\binom{2k-1}{k-1} + \binom{2k-1}{k-2}.$$

□

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Example: Suppose $G = O_2$ and $[5] = \{1,2,3,4,5\}$. Then
$V_2 = \{\{1,2\},\{1,3\},\{1,4\}\}$, $V_1 = \{1,5\}$, $V_0 = \{\{2,3\},\{2,4\},\{2,5\},\{3,4\},\{3,5\},\{4,5\}\}$ is an RDF for G. Also, $V_2 = \{\{1,2\},\{1,5\},\{2,5\},\{3,4\}\}$, $V_1 = \emptyset$, $V_0 = \{\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,5\},\{4,5\}\}$ is a $TRDF$ for G (see figure 2).

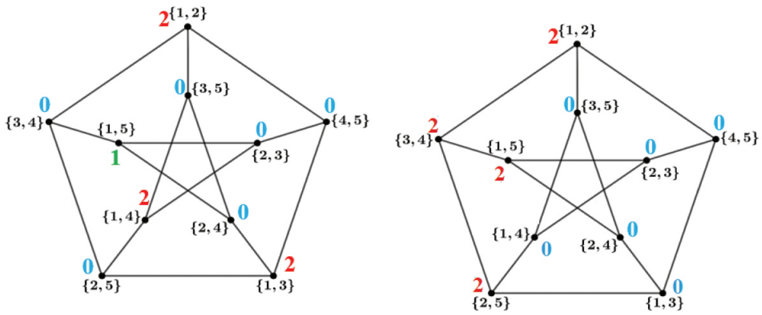

FIGURE 2. An RDF and a $TRDF$ for Petersen graph

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