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MAPPABLE NEARLY ORTHOGONAL ARRAYS USING HADAMARD MATRICES

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Abstract. Hadamard matrices play an important role in the construction of symmetric and asymmetric orthogonal arrays of various strengths. In this paper, a new method is proposed for the construction of mappable nearly orthogonal arrays (MNOAs) using Hadamard matrices. [14] used Mappable nearly orthogonal arrays for better space filling designs for computer experiments. The constructed MNOAs can able to accommodate more factors and have much better projection uniformity in comparison of existing MNOAs.

1. Introduction

Hadamard matrices are used in construction of combinatorial designs, such as balanced incomplete block designs and orthogonal arrays, which have applications in cryptography, and coding theory. The applications and constructions of Hadamard matrices are given in an excellent review paper of [3]. [8] discussed different methods of constructions and applications of Hadamard matrices in a monograph on Hadamard matrices in which they have displayed and summarized all known and unknown Hadamard matrices of different orders. In the past years, several researchers developed many methods for construction of symmetric and asymmetric orthogonal array of various strength. Symmetric and asymmetric orthogonal arrays have been widely used in scientific, agricultural and industrial investigations (see [7]). Some methods of generating orthogonal arrays are given by [2]. The construction of orthogonal arrays is an important problem in combinatorial designs which holds great significance for design of experiments in

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the statistical analysis. The construction methods of orthogonal arrays of various strengths and indexes and research problems in the area of orthogonal arrays are broadly given in [7].
The most popular use of orthogonal arrays is in space-filling designs for computer experiments. Space-filling designs are constructed by using orthogonal arrays and similar designs of orthogonal arrays, such as randomized orthogonal arrays see [5] and orthogonal array-based on Latin hypercubes (see [6]). [6] utilized orthogonal arrays of strength t to perceive t-dimensional space-filling property.

[9] derived some series of mappable nearly orthogonal arrays (MNOAs) using resolvable orthogonal arrays. In these arrays, each column of a group is orthogonal to a large proportion of the other columns, and these arrays are easily convertible to fully orthogonal arrays via a mapping of symbols in each column to a possibly smaller or same set of symbols. The importance and applications of this type of arrays have been of considerable interest because of their inherent better space-filling properties. [10] proposed two methods for the construction of column-orthogonal nearly strong orthogonal arrays. These arrays enjoy column-orthogonality and inherit the attractive two-dimensional space-filling property of strong orthogonal arrays. [11] derived orthogonal array based designs under a broad class of space- filling criteria. [14] constructed a series of MNOAs which possess near column-orthogonality, projection uniformity. [15] constructed a series of designs which possess both (near) column-orthogonality and projection uniformity, called (nearly) column-orthogonal mappable nearly orthogonal arrays (MNOAs).
[12] constructed new series of mappable nearly orthogonal arrays using difference schemes. [13] constructed nearly orthogonal arrays mappable into symmetric orthogonal arrays of strength two using Hadamard matrices. This paper proposes a new method for the construction of mappable nearly orthogonal arrays (MNOAs) using Hadamard matrices. The constructed MNOAs have achieved better degree of orthogonality. For example, the constructed $MNOA[144,((12)^{11})^7,(2^{11})^7]$, accommodates 77 factors at 12 levels achieves stratification of 86.84 % column pairs on 12×12 grid in 2577 out of 2926 two dimensions and on a 2×12 or 12×2 grid in the remaining 385 two dimensions. Similarly, the $MNOA[256,((16)^{15})^{17},(2^{15})^{17}]$ is constructed that accommodates 255 factors at 16 levels achieve stratification of 94.5% column pairs on 16×16 grid, while the MNOA constructed by [14] with same 256 runs accommodates only 170 factors with 94.7 % columns pairs. The MNOA constructed by [15] with 256 runs accommodates 204 factors with 94.5% column pairs that achieving stratification on 16×16 grid. So, our designs accommodate more factors then others. The MNOA with 144 runs is not constructed previously.

We give some preliminaries as Hadamard matrix, orthogonal array, and mappable nearly orthogonal array (MNOA) in Section 2. In Section 3, we present a method to construct some new mappable nearly orthogonal array. The proposed method is illustrated in Section 4 and we also provide Tables of constructed MNOAs. This paper is concluded in Section 5.

2. Preliminaries

Here, we give some important results and definitions that are useful in the paper.

2.1. **Hadamard matrix.** A Hadamard matrix is a square matrix of elements ones and minus ones whose row (and therefore column) vectors are orthogonal. The order N of such a matrix is necessarily 1,2 or a multiple of 4. Thus, for a Hadamard matrix H of order N,

$$HH^T = NI_N$$

where I_N is an identity matrix of order $N \times N$. Hadamard matrices of order 2 and 4, respectively, are given below:

$$H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

2.2. **Orthogonal array.** An $N \times n$ array B, with entries from a set of levels $S = (0, 1 \ldots, s-1)$ of $s(\geq 2)$ elements, is called a symmetric orthogonal array of strength t, run size N, n factors, each of s levels if every $N \times t$ submatrix of B contains all possible $1 \times t$ row vectors exactly λ times. The array B is denoted by $OA[N, n, s, t]$ and the number λ is called index of the array. For a symmetric orthogonal array $N = \lambda s^t$. For example;

$$OA(4,3,2,2) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The orthogonality in orthogonal arrays (OAs) is called combinatorial orthogonality. This type of orthogonality leads to t- and lower-dimensional projection uniformity properties. Another type of orthogonality is column-orthogonality, which means, the inner product of any two columns of the design is zero and such a design is called column-orthogonal design. When neither combinatorial nor column-orthogonality is possible, we look for near orthogonality. Throughout this paper, consider designs with combinatorial orthogonality.

2.3. **Mappable nearly orthogonal array.** A mappable nearly orthogonal array denoted as $MNOA$ $\left[N, \prod_{i=1}^{m} s_i^{c_i}, \prod_{i=1}^{m} \prod_{j=1}^{c_i} r_{ij}\right]$ is an $N \times \tilde{c}$ array whose $\tilde{c} = c_1 + c_2 + \cdots + c_m$ columns can be partitioned into m disjoint groups of $c_1, c_2, \ldots, c_m$, columns with the following properties:

I. for $i = 1, \ldots, m$, every column of the ith group is populated by s_i symbols;
II. any two columns from different groups are orthogonal;
III. for $i = 1, \ldots, m$ and for $j = 1, \ldots, c_i$ the s_i symbols in the jth column of the ith group can be mapped to a set of $r_{ij} \leq s_i$ symbols such that these mappings convert the array into an orthogonal array $OA[N, \prod_{i=1}^{m} \prod_{j=1}^{c_i} r_{ij}]$ of strength two.

In particular, if $s_i = s, c_i = c$ and $r_{ij} = r$ for every i and j, then a mappable nearly orthogonal array is denoted as $A = MNOA[N, (s^c)^m, (r^c)^m]$.

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By property II, in a mappable nearly orthogonal array before mapping, each of the c_i columns in the i group is orthogonal to at least a proportion $\tilde{\pi} = (\tilde{c} - c_i)/(\tilde{c} - 1)$ of the other columns. This leads to the following measures of the pre-mapping degree of orthogonality among the columns:

$$\overline{\pi} = \frac{\sum_{i=1}^{m} c_i \pi_i}{\sum_{i=1}^{m} c_i} = (\tilde{c}^2 - \sum_{i=1}^{m} c_i^2)/\{\tilde{c}(\tilde{c} - 1)\}$$

$$\pi_{min} = min_{1 \leq i \leq m} \pi_i = (\tilde{c} - max_{1 \leq i \leq m} c_i)/(\tilde{c} - 1)$$

if $c_1 = c_2 = \cdots = c_m = c$, then $\tilde{c} = mc$, where m and c are the number of groups and number of columns respectively.We have

$$\overline{\pi} = \pi_{min} = (m-1)c/(mc-1) \tag{2.1}$$

3. Method of Construction

Hadamard matrices is used in the construction of two symbol orthogonal arrays of strength two. The Foldover technique is used to obtain the even strength orthogonal arrays with two symbols from orthogonal arrays of odd strength. This result is originally derived by [1], see also [4]. Here, a new method is proposed to construct mappable nearly orthogonal arrays. The method of construction is described in the following steps;

Step I: Consider a normalized Hadamard matrix H of order $N = 2^k$, Replace -1 by 0 in the matrix H and delete the column that contains only 1 to obtain matrix C of order $N \times (N-1)$.

Step II: Let D_1 and D_2 denote the matrices C and $\overline{C}$ respectively where the matrix $\overline{C}$ can be obtained from the matrix C after replacing 0 and 1 by 1 and 0 respectively.

Step III: Replace the $l = N/2$ occurrences of each of the symbols $h = 0, 1$ in the ith $i = 1, 2, \ldots, (N-1)$ column of matrix D_λ, $\lambda = 1, 2$ by N symbols from the set $S = 0, 1, 2, \ldots, (N-1)$ to obtain matrix R_λ as follows:
Define

$$t_h = \{hl, hl+1, \ldots, hl + (l-1)\}, h = 0, 1 \tag{3.1}$$

and replace the k occurrences of symbol h by the $N/2$ members of t_h according to as given in (3.1), that is, the first value of h is replaced by hk and second value by $hk+1$ and so on. So that each column of R is a populated of $0, 1, \ldots, (N-1)$ symbols. Let $r_\lambda(0)$,$r_\lambda(1)$. . . ,$r_\lambda(N-1)$ denote the N rows of matrix R_λ.

Step IV: Consider an orthogonal array $B_\lambda = OA(\lambda N^2, M, N, 2)$, where $\lambda = 1$ or $\lambda = 2$. Divide M columns of the orthogonal array B_λ into M disjoint groups i.e. consider each column of B_λ as a group. Without loss of generality, let the N symbols in any column of B_λ be $0, 1, \ldots, (N-1)$.

Step V: Replace the ith, $i = 0, 1, \ldots, (N-1)$ symbol by the $r_\lambda(i)$ row of the matrix R_λ to get the pre-mapping or nearly orthogonal array with M groups, every group containing $N-1$ columns, where every column is populated by N symbols in S.

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Step VI: For obtaining post-mapping symmetric orthogonal array, do the revert mapping i.e. map the N symbols into two symbols 0 and 1 according to (3.1) as

$$hl, hl+1, \ldots, hl+(l-1) \to h = 0, 1 \tag{3.2}$$

Hence, the resultant array or post mapping array will be required symmetric orthogonal array of strength two. Thus, we have following result:

Theorem 3.1. *Existence of a Hadamard matrix of order N and orthogonal array $B_\lambda = OA(\lambda N^2, M, N, 2)$, $\lambda = 1$ or 2, implies the existence of mappable nearly orthogonal array $A = MNOA[\lambda N^2, (N^{(N-1)})^M, (2^{(N-1)})^M]$.*

Using above Theorem 3.1, the three types of construction are given below:

Construction I.
For $\lambda = 1, N = 2^k, k > 1$, a $MNOA[N^2, ((N^{(N-1)})^{(N+1)}, (2^{(N-1)})^{(N+1)}]$ can be constructed by using orthogonal array $B_1 = OA[N^2, N+1, N, 2]$ in Theorem 3.1.

Construction II.
For $\lambda = 2, N = 2^k, k > 1$, a $MNOA[4N^2, ((N^{(N-1)})^{(2N+1)}, (2^{(N-1)})^{(2N+1)}]$ can be constructed by using orthogonal array $B_1 = B_2 = OA[2N^2, 2N+1, N, 2]$ in Theorem **??**.

Construction III.
For $\lambda = 1, N \neq 2^k, k > 1$, a $MNOA[N^2, ((N^{(N-1)})^M, (2^{(N-1)})^M]$ can be constructed by using orthogonal array $B = OA[N^2, M, N, 2]$ in Theorem 3.1.

4. Illustration

Here, we illustrate the method and constructions through some examples:

Example 1 (Construction I)**.** *Let $\lambda = 1$ and $k = 2$. Consider a matrix H of order 4 after replacing element -1 by 0 as given*

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \tag{4.1}$$

Using Step I, delete the first column from matrix H in (4.1), to get the matrix D_1 of order 4×3

$$D_1 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_1 = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 3 & 0 \\ 3 & 0 & 1 \\ 1 & 1 & 3 \end{bmatrix} \tag{4.2}$$

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where for obtaining matrix R_1 using Step II, replace the occurrences of symbols 0 and 1 in each column of D_1 are replaced by set of symbols $S = (0, 1, 2, 3)$ according to (3.1). Let $r_1(0), r_1(1), r_1(2)$ and $r_1(3)$ are rows of matrix R_1. According to Step IV, consider orthogonal array $B_1 = OA[16, 5, 4, 2]$ as Using

TABLE 1. Orthogonal array $B_1 = OA[16, 5, 4, 2]$

row 1 to 8	row 9 to 16
0 0 0 0 0	2 0 2 3 1
0 1 1 1 1	2 1 3 2 0
0 2 2 2 2	2 2 0 1 3
0 3 3 3 3	2 3 1 0 2
1 0 1 2 2	3 0 3 1 2
1 1 0 3 2	3 1 2 0 3
1 2 3 0 1	3 2 1 3 0
1 3 2 1 0	3 3 0 2 1

Step V, divide the five columns of orthogonal array B_1 into five disjoint groups, every group consisting of four symbols of set $S = (0, 1, 2, 3)$. For obtaining the pre-mapping array the symbol i of each group is replaced by $r_1(i)$ row of matrix R_1. Hence the pre-mapping array contains five groups, each group has three columns and each column populated by symbols of set S is given in Table 2.

TABLE 2. Pre-mapping array of $MNOA[16, (4^3)^5, (2^3)^5]$

Group 1	Group 2	Group 3	Group 4	Group 5
2 2 2	2 2 2	2 2 2	2 2 2	2 2 2
2 2 2	0 3 0	0 3 0	0 3 0	0 3 0
2 2 2	3 0 1	3 0 1	3 0 1	3 0 1
2 2 2	1 1 3	1 1 3	1 1 3	1 1 3
0 3 0	2 2 2	0 3 0	3 0 1	1 1 3
0 3 0	0 3 0	2 2 2	1 1 3	3 0 1
0 3 0	3 0 1	1 1 3	2 2 2	0 3 0
0 3 0	1 1 3	3 0 1	0 3 0	2 2 2
3 0 1	2 2 2	3 0 1	1 1 3	0 3 0
3 0 1	0 3 0	1 1 3	3 0 1	2 2 2
3 0 1	3 0 1	2 2 2	0 3 0	1 1 3
3 0 1	1 1 3	0 3 0	2 2 2	3 0 1
1 1 3	2 2 2	1 1 3	0 3 0	3 0 1
1 1 3	0 3 0	3 0 1	2 2 2	1 1 3
1 1 3	3 0 1	0 3 0	1 1 3	2 2 2
1 1 3	1 1 3	2 2 2	3 0 1	0 3 0

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Using Step VI, do the revert mapping as (3.2) means the symbols of set $S = (0, 1, 2, 3)$ are mapped to $h = (0, 1)$ symbols, to get post-mapping arrays which is symmetric orthogonal array of strength two displayed in Table 3.

TABLE 3. Post-mapping array of $MNOA[16, (4^3)^5, (2^3)^5]$

Group 1	Group 2	Group 3	Group 4	Group 5
1 1 1	1 1 1	1 1 1	1 1 1	1 1 1
1 1 1	0 1 0	0 1 0	0 1 0	0 1 0
1 1 1	1 0 0	1 0 0	1 0 0	1 0 0
1 1 1	0 0 1	0 0 1	0 0 1	0 0 1
0 1 0	1 1 1	0 1 0	1 0 0	0 0 1
0 1 0	0 1 0	1 1 1	0 0 1	1 0 0
0 1 0	1 0 0	0 0 1	1 1 1	0 1 0
0 1 0	0 0 1	1 0 0	0 1 0	1 1 1
1 0 0	1 1 1	1 0 0	0 0 1	0 1 0
1 0 0	0 1 0	0 0 1	1 0 0	1 1 1
1 0 0	1 0 0	1 1 1	0 1 0	0 0 1
1 0 0	0 0 1	0 1 0	1 1 1	1 0 0
0 0 1	1 1 1	0 0 1	0 1 0	1 0 0
0 0 1	0 1 0	1 0 0	1 1 1	0 0 1
0 0 1	1 0 0	0 1 0	0 0 1	1 1 1
0 0 1	0 0 1	1 1 1	1 0 0	0 1 0

Example 2 (Construction II)**.** *Let $\lambda = 2$ and $k = 2$. Consider the matrix H of order 4. Using Step I, delete the first column from the matrix H containing only 1 elements and obtain the two matrices D_1 and D_2 are given in (4.3).*

$$D_1 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \tag{4.3}$$

Following Step III, replace all the occurrences of symbols 0 and 1 from the set $S = (0, 1, 2, 3)$ in the matrix D_1 and D_2, to get the matrix R_λ, $\lambda = 1, 2$. R_1 and R_2 matrices are displayed in (2).

$$R_1 = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 3 & 0 \\ 3 & 0 & 1 \\ 1 & 1 & 3 \end{bmatrix} \tag{4.4}$$

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$$R_2 = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$

Consider the orthogonal arrays $B_1 = B_2 = OA(32, 9, 4, 2)$ given in Table 4.

TABLE 4. Orthogonal array of B_1=B_2= $OA(32, 9, 4, 2)$

Row 1 to 16	Row 17 to 32
0 0 0 0 0 0 0 0 0	2 2 2 2 2 2 2 2 0
0 1 2 3 0 1 2 3 1	2 3 0 1 2 3 0 1 1
0 2 0 2 3 1 3 1 2	2 0 2 0 1 3 1 3 2
0 3 2 1 3 0 1 2 3	2 1 0 3 1 2 3 0 3
0 0 3 3 2 2 1 1 0	2 2 1 1 0 0 3 3 0
0 1 1 0 2 3 3 2 1	2 3 3 2 0 1 1 0 1
0 2 3 1 1 3 2 0 2	2 0 1 3 3 1 0 2 2
0 3 1 2 1 2 0 3 3	2 1 3 0 3 0 2 1 3
1 1 1 1 1 1 1 1 0	3 3 3 3 3 3 3 3 0
1 0 3 2 1 0 3 2 1	3 2 1 0 3 2 1 0 1
1 3 1 3 2 0 2 0 2	3 1 3 1 0 2 0 2 2
1 2 3 0 2 1 0 3 3	3 0 1 2 0 3 2 1 3
1 1 2 2 3 3 0 0 0	3 3 0 0 1 1 2 2 0
1 0 0 1 3 2 2 3 1	3 2 2 3 1 0 0 1 1
1 3 2 0 0 2 3 1 2	3 1 0 2 2 0 1 3 2
1 2 0 3 0 3 1 2 3	3 0 2 1 2 1 3 0 3

Write $A = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$, where B_1 and B_2 both are same orthogonal arrays. The nine columns of array B_λ, $\lambda = 1, 2$ are divided into nine groups i.e, each column of array B_λ is considered as a group and every group of array B_λ contains four symbols $0, 1, 2, 3$. For obtaining the pre-mapping array is displayed in Table 8 and Table 9 in Appendix A, the symbols of every group of array B_λ, $\lambda = 1, 2$, are replaced by rows $r_\lambda(i)$,$i = 0, 1, 2, 3$ of matrix R_λ according to Step V.
Using Step VI, do the revert mapping as (3.1) means that set of symbols $S = (0, 1, 2, 3)$ mapped to $h = (0, 1)$ symbols, to get the resultant post-mapping arrays is symmetric orthogonal array of strength two displayed in Table 10 and Table 11 in Appendix A. Hence, the mappable nearly orthogonal array $MNOA[64, (4^3)^9, (2^3)^9]$ is constructed.

The bivariate projection between two columns from different Groups display in Figure 1, into 4×4 grid in Appendix A. Similarly, in Figure 2, projection of different Groups into 2×2 grid and Figure 3, projection between two column of same Group into 2×2 grid of pre-mapping array given in Table 2.

TABLE 5. Some constructed MNOA based on construction I

k	N	$B = OA[N^2, N+1, N, 2]$	A	$\overline{\pi}$
2	4	$B = OA[16, 5, 4, 2]$	$MNOA[16, (4^3)^5, (2^3)^5)]$	0.8571
3	8	$B = OA[64, 9, 8, 2]$	$MNOA[64, (8^7)^9, (2^7)^9]$	0.9032
4	16	$B = OA[256, 17, 16, 2]$	$MNOA[256, ((16)^{15})^{17}, (2^{15})^{17})]$	0.9448
5	32	$B = OA[1024, 33, 32, 2]$	$MNOA[1024, ((32)^{31})^{33}(2^{31})^{33})]$	0.9706
6	64	$B = OA[4096, 65, 64, 2]$	$MNOA[4096, ((64)^{63})^{65}, (2^{63})^{65})]$	0.9848
7	128	$B = OA[16384, 129, 128, 2]$	$MNOA[16384, ((128)^{127})^{129}, (2^{127})^{129})]$	0.9919

TABLE 6. Some constructed MNOA based on construction II

k	N	$B = OA[N^2, N+1, N, 2]$	A	$\overline{\pi}$
2	4	$B = OA[32, 9, 4, 2]$	$MNOA[64, (4^3)^9, (2^3)^9)]$	0.9230
3	8	$B = OA[128, 17, 8, 2]$	$MNOA[256, (8^7)^{17}, (2^7)^{17}]$	0.9491
4	16	$B = OA[512, 33, 16, 2]$	$MNOA[1024, \{(16)^{15}\}^{33}, \{(2)^{15}\}^{33})]$	0.9716
5	32	$B = OA[2048, 65, 32, 2]$	$MNOA[4096, \{(32)^{31}\}^{65}, \{(2)^{31}\}^{65})]^*$	0.9851
6	64	$B = OA[8192, 129, 64, 2]$	$MNOA[16384, ((64)^{63})^{129}, (2^{63})^{129})]$	0.9848

Similarly, one can obtain many more symmetric mappable nearly orthogonal arrays of strength two, some of them are listed in Table 5 and Table 6 using construction I and II.
Table 7, displays some mappable nearly orthogonal arrays obtained using construction III for some non-prime N. The Hadamard matrices, arrays B and constructed MNOAs are not displayed here because of their big size. These are available with authors.

TABLE 7. Some constructed MNOA based on construction III

N	$B = OA[N^2, N+1, N, 2]$	A	$\overline{\pi}$
12	$B = OA[144, 7, 12, 2]$	$MNOA[144, (12^{11})^7, (2^{11})^7)]$	0.8684
12	$B = OA[288, 12, 12, 2]$	$MNOA[288, (12^{11})^{12}, (2^{11})^{12}]$	0.9236

5. Conclusion

This paper proposes a new method to construct MNOA using Hadamard matrices. The post- mapping array of these MNOAs is fully orthogonal array of strength two. The constructed MNOAs are tabulated in Tables 5, 6 and 7 along with corresponding values of degree of orthogonality. It is observed that the constructed MNOAs have high value of degree of orthogonality. It is also observed, that the proposed method can also be applied to construct MNOAs for the values of N where $N \neq 2^k$.

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References

[1] G. E. P. Box and K. B. Wilson, On the experimental attainment of optimum condition, *J. Roy. Statist. Soc. Ser. B*, **23** (1951) 1–38.

[2] R. C. Bose and K. A. Bush, Orthogonal arrays of strength two and three,*Ann. Math. Statistics*, **23** (1952) 508–524.

[3] A. S. Hedayat and W. D. Wallis, Hadamard matrices and their applications, *Ann. Statist.*, **6** no. 6 (1978) 1184–1238.

[4] A. S. Hedayat and J. Stufken, Optimum design and analysis of experiments, *Elsevier Science Publishers B. V.*, (1988) 47–58.

[5] A. B. Owen, Orthogonal arrays for computer experiments, integration and visualization, *Statist. Sinica*, **2** no. 2 (1992) 439–452.

[6] B. Tang, Orthogonal array-based Latin hypercubes *J. Amer. Statist. Assoc.*, **88** no. 424 (1993) 1392–1397.

[7] A. S. Hedayat, N. J. A. Sloane and J. Stufken,*Orthogonal arrays. Theory and applications*, Springer-Verlag, New York, 1999.

[8] V. K. Gupta, A. Dhandapani, R. Parsad and W. F. Kuhfeld, Monograph review: Hadamard Matrices, *J. Stat. Theory Pract.*, **2** no. 4 (2008) 713–714.

[9] R. Mukerjee, F. Sun and B. Tang, Nearly orthogonal arrays mappable into fully orthogonal arrays, *Biometrika*, **101** no. 4 (2014) 957–963.

[10] W. Li, M. Q. Liu and J. F. Yang, Column-orthogonality nearly strong orthogonal arrays, *J. Statist. Plann. Inference*, **215** (2021) 184–192.

[11] G. Chen and B. Tang, A study of orthogonal array-based designs under a broad class of space-filling criteria, *Ann. Statist.*, **50** no.5 (2022) 2925–2949.

[12] P. Singh, M. D. Mazumder and S. Babu, Construction of nearly orthogonal arrays mappable into fully orthogonal arrays of strength two and three, *International Journal of Mathematics and Statistics*, **24** no. 1 (2023) 37–50.

[13] P. Singh, M. D. Mazumder and S. Babu, Nearly orthogonal arrays mappable into symmetric orthogonal arrays of strength two, *International Journal of Statistics and Reliability Engineering*, **10** no. 2 (2023) 431–437.

[14] W. Li, M. Q. Liu and J. F. Yang, Several new classes of space-filling designs, *Statist. Papers*, **65** no. 1 (2024) 357–379.

[15] H. Liu, F. Sun, D. K. Lin and M. Q. Liu, On construction of mappable nearly orthogonal array with Column-orthogonality, *Commun. Math. Stat.*, **13** no. 2 (2025) 455–480.

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6. Appendix A

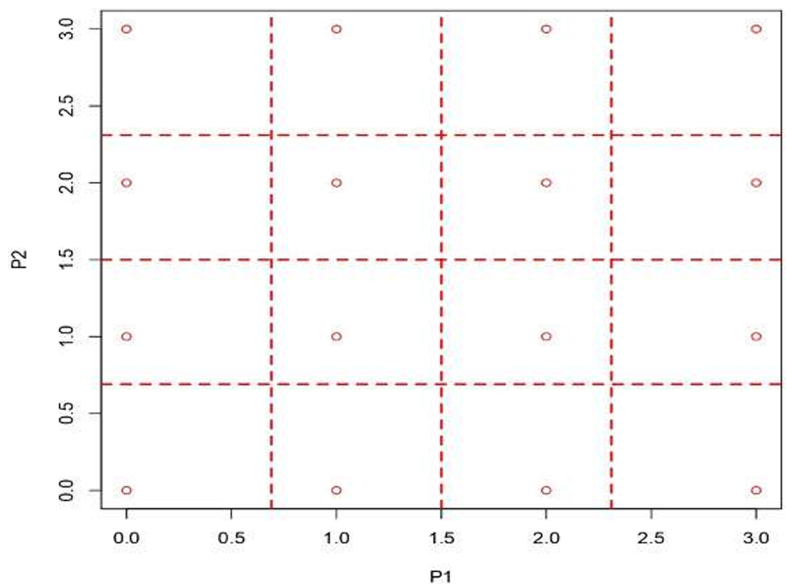

FIGURE 1. Bivariate projection between two columns of different groups in 4×4 grid

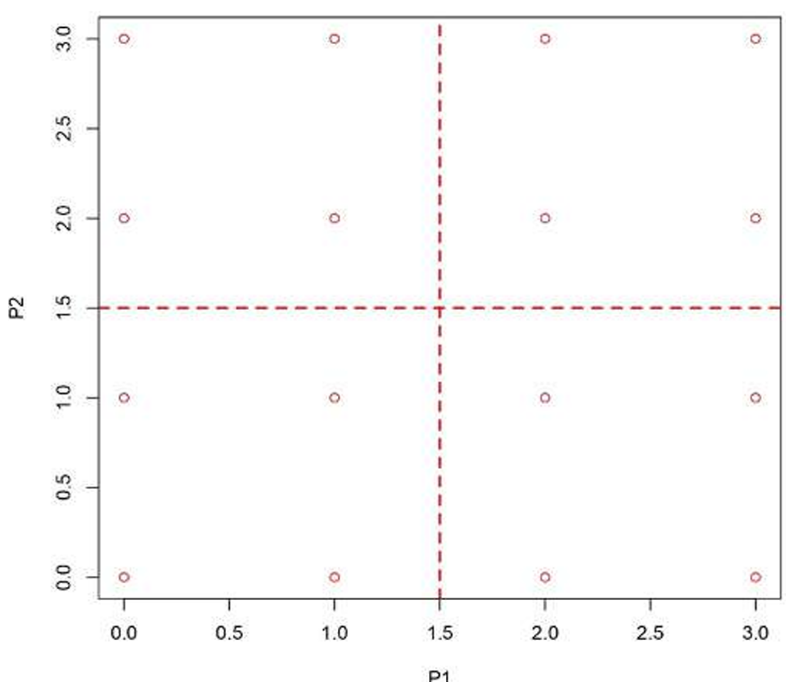

FIGURE 2. Bivariate projection between two columns of different groups in 2×2 grid

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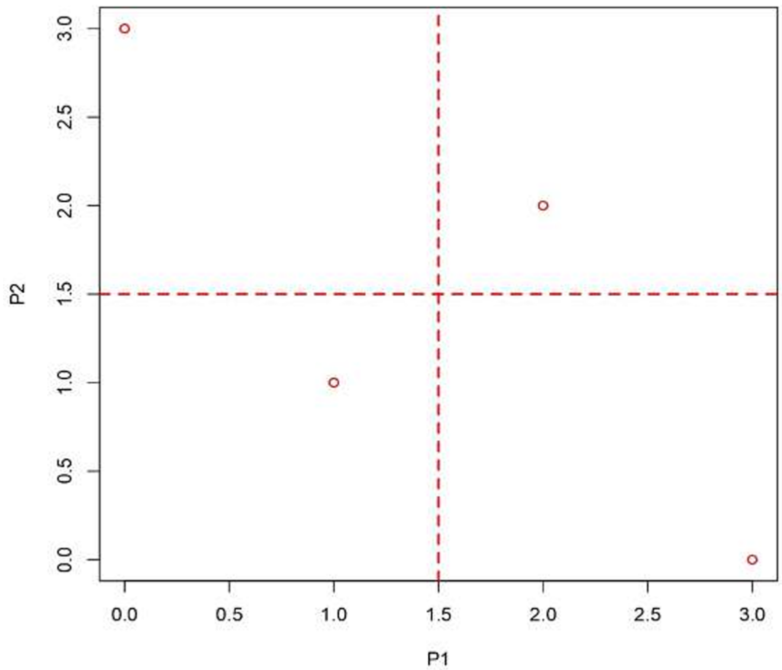

FIGURE 3. Bivariate projection between two columns of same group in 2×2 grid

TABLE 8. Pre-mapping array of $MNOA[64, (4^3)^9, (2^3)^9]$

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8	Group 9
2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2
2 2 2	3 0 1	1 1 3	1 1 3	2 2 2	0 3 0	3 0 1	1 1 3	0 3 0
2 2 2	2 2 2	3 0 1	3 0 1	1 1 3	0 3 0	1 1 3	0 3 0	3 0 1
2 2 2	3 0 1	0 3 0	0 3 0	1 1 3	2 2 2	0 3 0	3 0 1	1 1 3
2 2 2	1 1 3	1 1 3	1 1 1	3 0 1	3 0 1	0 3 0	0 3 0	2 2 2
2 2 2	0 3 0	2 2 2	2 2 2	3 0 1	1 1 3	1 1 3	3 0 1	0 3 0
2 2 2	1 1 3	0 3 0	0 3 0	0 3 0	1 1 3	3 0 1	2 2 2	3 0 1
2 2 2	0 3 0	3 0 1	3 0 1	0 3 0	3 0 1	2 2 2	1 1 3	1 1 3
0 3 0	0 3 0	0 3 0	0 3 0	0 3 0	0 3 0	0 3 0	0 3 0	2 2 2
0 3 0	1 1 3	3 0 1	3 0 1	0 3 0	2 2 2	1 1 3	3 0 1	0 3 0
0 3 0	0 3 0	1 1 3	1 1 3	3 0 1	2 2 2	3 0 1	2 2 2	3 0 1
0 3 0	1 1 3	2 2 2	2 2 2	3 0 1	0 3 0	2 2 2	1 1 3	1 1 3
0 3 0	3 0 1	3 0 1	3 0 1	1 1 3	1 1 3	2 2 2	2 2 2	2 2 2
0 3 0	2 2 2	0 3 0	0 3 0	1 1 3	3 0 1	3 0 1	1 1 3	0 3 0
0 3 0	3 0 1	2 2 2	2 2 2	2 2 2	3 0 1	1 1 3	0 3 0	3 0 1
0 3 0	2 2 2	1 1 3	1 1 3	2 2 2	1 1 3	0 3 0	3 0 1	1 1 3
3 0 1	3 0 1	3 0 1	3 0 1	3 0 1	3 0 1	3 0 1	3 0 1	2 2 2
3 0 1	2 2 2	0 3 0	0 3 0	3 0 1	1 1 3	2 2 2	0 3 0	0 3 0
3 0 1	3 0 1	2 2 2	2 2 2	0 3 0	1 1 3	0 3 0	1 1 3	3 0 1
3 0 1	2 2 2	1 1 3	1 1 3	0 0 3	3 0 1	1 1 3	2 2 2	1 1 3

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TABLE 9. Table 8 Continue· · ·

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8	Group 9
3 0 1	0 3 0	0 3 0	0 3 0	2 2 2	2 2 2	1 1 3	1 1 3	2 2 2
3 0 1	1 1 3	3 0 1	3 0 1	2 2 2	0 3 0	0 3 0	2 2 2	0 3 0
3 0 1	0 3 0	1 1 3	1 1 3	1 1 3	0 3 0	2 2 2	3 0 1	3 0 1
3 0 1	1 1 3	2 2 2	2 2 2	1 1 3	2 2 2	3 0 1	0 3 0	1 1 3
1 1 3	1 1 3	1 1 3	1 1 3	1 1 3	1 1 3	1 1 3	1 1 3	2 2 2
1 1 3	0 3 0	2 2 2	2 2 2	1 1 3	3 0 1	0 3 0	2 2 2	0 3 0
1 1 3	1 1 3	0 3 0	0 3 0	2 2 2	3 0 1	2 2 2	3 0 1	3 0 1
1 1 3	0 3 0	3 0 1	3 0 1	2 2 2	1 1 3	3 0 1	0 3 0	1 1 3
1 1 3	2 2 2	2 2 2	2 2 2	0 3 0	0 3 0	3 0 1	3 0 1	2 2 2
1 1 3	3 0 1	1 1 3	1 1 3	0 3 0	2 2 2	2 2 2	0 3 0	0 3 0
1 1 3	2 2 2	3 0 1	3 0 1	3 0 1	2 2 2	0 3 0	1 1 3	3 0 1
1 1 3	3 0 1	0 3 0	0 3 0	3 0 1	0 3 0	1 1 3	0 3 0	1 1 3
0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
0 0 0	2 1 2	1 2 3	3 3 1	0 0 0	2 1 2	1 2 3	3 3 1	2 1 2
0 0 0	1 2 3	0 0 0	1 2 3	3 3 1	2 1 2	3 3 1	2 1 2	1 2 3
0 0 0	3 3 1	1 2 3	2 1 2	3 3 1	0 0 0	2 1 2	1 2 3	3 3 1
0 0 0	0 0 0	3 3 1	3 3 1	1 2 3	1 2 3	2 1 2	2 1 2	0 0 0
0 0 0	2 1 2	2 1 2	0 0 0	1 2 3	3 3 1	3 3 1	1 2 3	2 1 2
0 0 0	1 2 3	3 3 1	2 1 2	2 1 2	3 3 1	1 2 3	0 0 0	1 2 3
0 0 0	3 3 1	2 1 2	1 2 3	2 1 2	1 2 3	0 0 0	3 3 1	3 3 1
2 1 2	2 1 2	1 2 3	1 2 3	3 3 1	3 3 1	0 0 0	0 0 0	0 0 0
2 1 2	0 0 0	0 0 0	2 1 2	3 3 1	1 2 3	1 2 3	3 3 1	2 1 2
2 1 2	3 3 1	1 2 3	0 0 0	0 0 0	1 2 3	3 3 1	2 1 2	1 2 3
2 1 2	1 2 3	0 0 0	3 3 1	0 0 0	3 3 1	2 1 2	1 2 3	3 3 1
1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	0 0 0
1 2 3	3 3 1	0 0 0	2 1 2	1 2 3	3 3 1	0 0 0	2 1 2	2 1 2
1 2 3	0 0 0	1 2 3	0 0 0	2 1 2	3 3 1	2 1 2	3 3 1	1 2 3
1 2 3	2 1 2	0 0 0	3 3 1	2 1 2	1 2 3	3 3 1	0 0 0	3 3 1
1 2 3	1 2 3	2 1 2	2 1 2	0 0 0	0 0 0	3 3 1	3 3 1	0 0 0
1 2 3	3 3 1	3 3 1	1 2 3	0 0 0	2 1 2	2 1 2	0 0 0	2 1 2
1 2 3	0 0 0	2 1 2	3 3 1	3 3 1	2 1 2	0 0 0	1 2 3	1 2 3
1 2 3	2 1 2	3 3 1	0 0 0	3 3 1	0 0 0	1 2 3	2 1 2	3 3 1
3 3 1	3 3 1	3 3 1	3 3 1	3 3 1	3 3 1	3 3 1	3 3 1	0 0 0
3 3 1	1 2 3	2 1 2	0 0 0	3 3 1	1 2 3	2 1 2	0 0 0	2 1 2
3 3 1	0 0 0	3 3 1	2 1 2	0 0 0	1 2 3	0 0 0	1 2 32	1 2 3
3 3 1	2 1 2	2 1 2	1 2 3	0 0 0	3 3 1	1 2 3	2 1 2	3 3 1
3 3 1	3 3 1	0 0 0	0 0 0	2 1 2	2 1 2	1 2 3	1 2 3	0 0 0
3 3 1	1 2 3	1 2 3	3 3 1	2 1 2	0 0 0	0 0 0	2 1 2	2 1 2
3 3 1	0 0 0	0 0 0	1 2 3	1 2 3	0 0 0	2 1 2	3 3 1	1 2 3
3 3 1	2 1 2	1 2 3	2 1 2	1 2 3	2 1 2	3 3 1	2 2 2	3 3 1

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TABLE 10. Post-mapping array of $MNOA[64, (4^3)^9, (2^3)^9]$

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8	Group 9
1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1
1 1 1	1 0 0	0 0 1	0 0 1	1 1 1	0 1 0	1 0 0	0 0 1	0 1 0
1 1 1	1 1 1	1 0 0	1 0 0	0 0 1	0 1 0	0 0 1	0 1 0	1 0 0
1 1 1	1 0 0	0 1 0	0 1 0	0 0 1	1 1 1	0 1 0	1 0 0	0 0 1
1 1 1	0 0 1	0 0 1	0 0 0	1 0 0	1 0 0	0 1 0	0 1 0	1 1 1
1 1 1	0 1 0	1 1 1	1 1 1	1 0 0	0 0 1	0 0 1	1 0 0	0 1 0
1 1 1	0 0 1	0 1 0	0 1 0	0 1 0	0 0 1	1 0 0	1 1 1	1 0 0
1 1 1	0 1 0	1 0 0	1 0 0	0 1 0	1 0 0	1 1 1	0 0 1	0 0 1
0 1 0	0 1 0	0 1 0	0 1 0	0 1 0	0 1 0	0 1 0	0 1 0	1 1 1
0 1 0	0 0 1	1 0 0	1 0 0	0 1 0	1 1 1	0 0 1	1 0 0	0 1 0
0 1 0	0 1 0	0 0 1	0 0 1	1 0 0	1 1 1	1 0 0	1 1 1	1 0 0
0 1 0	0 0 1	1 1 1	1 1 1	1 0 0	0 1 0	1 1 1	0 0 1	0 0 1
0 1 0	1 0 0	1 0 0	1 0 0	0 0 1	0 0 1	1 1 1	1 1 1	1 1 1
0 1 0	1 1 1	0 1 0	0 1 0	0 0 1	1 0 0	1 0 0	0 0 1	0 1 0
0 1 0	1 0 0	1 1 1	1 1 1	1 1 1	1 0 0	0 0 1	0 1 0	1 0 0
0 1 0	1 1 1	0 0 1	0 0 1	1 1 1	0 0 1	0 1 0	1 0 0	0 0 1
1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 0 0	1 1 1
1 0 0	1 1 1	0 1 0	0 1 0	1 0 0	0 0 1	1 1 1	0 1 0	0 1 0
1 0 0	1 0 0	1 1 1	1 1 1	0 1 0	0 0 1	0 1 0	0 0 1	1 0 0
1 0 0	1 1 1	0 0 1	0 0 1	0 0 1	1 0 0	0 0 1	1 1 1	0 0 1
1 0 0	0 1 0	0 1 0	0 1 0	1 1 1	1 1 1	0 0 1	0 0 1	1 1 1
1 0 0	0 0 1	1 0 0	1 0 0	1 1 1	0 1 0	0 1 0	1 1 1	0 1 0
1 0 0	0 1 0	0 0 1	0 0 1	0 0 1	0 1 0	1 1 1	1 0 0	1 0 0
1 0 0	0 0 1	1 1 1	1 1 1	0 0 1	1 1 1	1 0 0	0 1 0	0 0 1
0 0 1	0 0 1	0 0 1	0 0 1	0 0 1	0 0 1	0 0 1	0 0 1	1 1 1
0 0 1	0 1 0	1 1 1	1 1 1	0 0 1	1 0 0	0 1 0	1 1 1	0 1 0
0 0 1	0 0 1	0 1 0	0 1 0	1 1 1	1 0 0	1 1 1	1 0 0	1 0 0
0 0 1	0 1 0	1 0 0	1 0 0	1 1 1	0 0 1	1 0 0	0 1 0	0 0 1
0 0 1	1 1 1	1 1 1	1 1 1	0 1 0	0 1 0	1 0 0	1 0 0	1 1 1
0 0 1	1 0 0	0 0 1	0 0 1	0 1 0	1 1 1	1 1 1	0 1 0	0 1 0
0 0 1	1 1 1	1 0 0	1 0 0	1 0 0	1 1 1	0 1 0	0 0 1	1 0 0
0 0 1	1 0 0	0 1 0	0 1 0	1 0 0	0 1 0	0 0 1	0 1 0	0 0 1
0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
0 0 0	1 0 1	0 1 1	1 1 0	0 0 0	1 0 1	0 1 1	1 1 0	1 0 1
0 0 0	0 1 1	0 0 0	0 1 1	1 1 0	1 0 1	1 1 0	1 0 1	0 1 1
0 0 0	1 1 0	0 1 1	1 0 1	1 1 0	0 0 0	1 0 1	0 1 1	1 1 0
0 0 0	0 0 0	1 1 0	1 1 0	0 1 1	0 1 1	1 0 1	1 0 1	0 0 0
0 0 0	1 0 1	1 0 1	0 0 0	0 1 1	1 1 0	1 1 0	0 1 1	1 0 1
0 0 0	0 1 1	1 1 0	1 0 1	1 0 1	1 1 0	0 1 1	0 0 0	0 1 1
0 0 0	1 1 0	1 0 1	0 1 1	1 0 1	0 1 1	0 0 0	1 1 0	1 1 0

TABLE 11. Table 10 Continue···

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8	Group 9
1 0 1	1 0 1	1 0 1	1 0 1	1 0 1	1 0 1	1 0 1	1 0 1	0 0 0
1 0 1	0 0 0	1 1 0	0 1 1	1 0 1	0 0 0	1 1 0	0 1 1	1 0 1
1 0 1	1 1 0	1 0 1	1 1 0	0 1 1	0 0 0	0 1 1	0 0 0	0 1 1
1 0 1	0 1 1	1 1 0	0 0 0	0 1 1	1 0 1	0 0 0	1 1 0	1 1 0
1 0 1	1 0 1	0 1 1	0 1 1	1 1 0	1 1 0	0 0 0	0 0 0	0 0 0
1 0 1	0 0 0	0 0 0	1 0 1	1 1 0	0 1 1	0 1 1	1 1 0	1 0 1
1 0 1	1 1 0	0 1 1	0 0 0	0 0 0	0 1 1	1 1 0	1 0 1	0 1 1
1 0 1	0 1 1	0 0 0	1 1 0	0 0 0	1 1 0	1 0 1	0 1 1	1 1 0
0 1 1	0 1 1	0 1 1	0 1 1	0 1 1	0 1 1	0 1 1	0 1 1	0 0 0
0 1 1	1 1 0	0 0 0	1 0 1	0 1 1	1 1 0	0 0 0	1 0 1	1 0 1
0 1 1	0 0 0	0 1 1	0 0 0	1 0 1	1 1 0	1 0 1	1 1 0	0 1 1
0 1 1	1 0 1	0 0 0	1 1 0	1 0 1	0 1 1	1 1 0	0 0 0	1 1 0
0 1 1	0 1 1	1 0 1	1 0 1	0 0 0	0 0 0	1 1 0	1 1 0	0 0 0
0 1 1	1 1 0	1 1 0	0 1 1	0 0 0	1 0 1	1 0 1	0 0 0	1 0 1
0 1 1	0 0 0	1 0 1	1 1 0	1 1 0	1 0 1	0 0 0	0 1 1	0 1 1
0 1 1	1 0 1	1 1 0	0 0 0	1 1 0	0 0 0	0 1 1	1 0 1	1 1 0
1 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0	0 0 0
1 1 0	0 1 1	1 0 1	0 0 0	1 1 0	0 1 1	1 0 1	0 0 0	1 0 1
1 1 0	0 0 0	1 1 0	1 0 1	0 0 0	0 1 1	0 0 0	0 1 1	0 1 1
1 1 0	1 0 1	1 0 1	0 1 1	0 0 0	1 1 0	0 1 1	1 0 1	1 1 0
1 1 0	1 1 0	0 0 0	0 0 0	1 0 1	1 0 1	0 1 1	0 1 1	0 0 0
1 1 0	0 1 1	0 1 1	1 1 0	1 0 1	0 0 0	0 0 0	1 0 1	1 0 1
1 1 0	0 0 0	0 0 0	0 1 1	0 1 1	0 0 0	1 0 1	1 1 0	0 1 1
1 1 0	1 0 1	0 1 1	1 0 1	0 1 1	1 0 1	1 1 0	1 1 1	1 1 0

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