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ON THE REFLEXIVE EDGE STRENGTH IN ZIGZAG GRAPHS

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Abstract. Let G be a simpple graph. The total k-labeling of G is the assignments of a non-negative integer from the set $\{0, 2, \ldots, 2\lfloor k/2 \rfloor\}$ to the vertices and a positive integer from the set $\{1, 2, \ldots, k\}$ to the edges of a graph G. It is called an edge irregular reflexive k-labeling of the graph G if the weights of any two different edges are distinct, where the edge weight is the sum of the label of the edge itself and the labels of its two end vertices. The minimum value k for which the graph G has an edge irregular reflexive k-labeling is called the reflexive edge strength of G. In this paper, we determine the exact value of the reflexive edge strength for the zigzag graph Z_m^n, where $n \geq 2$ and $m \geq 3$.

1. Introduction

Over the past 60 years, graph labeling has experienced rapid development, resulting in over 200 graph labeling methods that have been explored in more than 3000 papers, see Gallian [10]. This captivating field holds great potential and finds application in diverse areas such as astronomy, communication networks, circuit design, coding theory, radar technology, X-ray crystallography, and more. In the following, all graphs considered are finite, undirected, and simple.

In 1988, Chartrand et al. [9] introduced the concept of irregular labeling to challenge the notion that a simple graph is impossible to become irregular (which is possible only in multigraphs) and

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they proved that it is achievable in the simple graph by replacing the multiple edges incident on every vertex of a multigraph with a positive integer set on the edges of a simple graph of order at least 3. The minimum of maximal values is known as the irregularity strength of the graph G, denoted as $s(G)$ [9].

Moreover, Bača et al. [6] proposed irregular total labelings, namely an edge irregular total labeling and a vertex irregular total labeling, which consider the weights of edges and vertices, respectively. For a comprehensive survey of the latest and most relevant studies on irregular labeling and irregular total labelings, please refer to Gallian [10].

The following definition is taken from [6].

Definition 1. [6] For a graph $G = (V, E)$, a labeling $\xi : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ is a total k-labeling. The total k-labeling is an edge irregular total k-labeling if every two different edges xy and $x'y'$ have distinct weights, $wt_\xi(xy) \neq wt_\xi(x'y')$, where $wt_\xi(xy) = \xi(x) + \xi(xy) + \xi(y)$. Likewise, the total k-labeling is called a vertex irregular total k-labeling if every two different vertices x and y have the distinct weights, $wt_\xi(x) \neq wt_\xi(y)$, where $wt_\xi(x) = \xi(x) + \sum_{xy\in E(G)} \xi(xy)$. The minimum k for which such labelings exist is called the total edge irregularity strength of G, denoted by $\mathrm{tes}(G)$ (resp. the total vertex irregularity strength of G, denoted by $\mathrm{tvs}(G)$).

Inspired by the concepts of irregular labeling and irregular total labelings, Ryan et al. [15] introduced the concept of edge irregular reflexive labeling, where the vertex labels are represented as the vertex degrees contributed by the loops. They made two observations: (a) the vertex labels are non-negative even integers, which indicate that each loop contributes twice to the vertex degree, and (b) the vertex label 0 is allowed to represent a vertex without any loops [15].

Definition 2. [16] A total k-labeling f is a combination of an edge labeling $f_e : E(G) \to \{1, 2, \ldots, k_e\}$ and a vertex labeling $f_v : V(G) \to \{0, 2, \ldots, 2k_v\}$, where $k = \max\{k_e, 2k_v\}$. The total k-labeling f is called an *edge irregular reflexive k-labeling* of G if any two different edges xy and $x'y'$ of G have the distinct edge weights $wt_f(xy) \neq wt_f(x'y')$, where $wt_f(xy) = f_v(x) + f_e(xy) + f_v(y)$. The smallest value k for such a labeling exists is called the reflexive edge strength of G, denoted by $\mathrm{res}(G)$.

Furthermore, Tanna et al. [16] also proved a lower bound for the reflexive edge strength of any graph G as follows.

Lemma 1. [16] *For a graph G,*

$$\mathrm{res}(G) \geq \begin{cases} \left\lceil \frac{|E(G)|}{3} \right\rceil, & \text{if } |E(G)| \not\equiv 2, 3 \pmod 6, \\ \left\lceil \frac{|E(G)|}{3} \right\rceil + 1, & \text{if } |E(G)| \equiv 2, 3 \pmod 6. \end{cases}$$

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Bača et al. [4] provided a conjecture for the reflexive edge strength of any graph. Additionally, they determined the exact value of the reflexive edge strength for generalized friendship graphs in [4], cycles, the Cartesian product of two cycles, and the join of a path or a cycle with $2K_1$ in [5].

Conjecture 1. [4] *Any graph G with maximum degree $\Delta(G)$ satisfies*

$$\operatorname{res}(G) = \max\left\{ \left\lceil \frac{|E(G)|}{3} + r \right\rceil, \left\lfloor \frac{\Delta+2}{2} \right\rfloor \right\}$$

where $r = 1$ for $|E(G)| \equiv 2, 3 \pmod 6$, and zero otherwise.

In addition, Basher studied an edge irregular reflexive labeling of circulant graphs in [7] and toroidal fullerence in [8]. Yoong et al. investigated the exact values of the reflexive edge strength for several graphs, including the corona product of two paths and corona product of a path with isolated vertices in [17], certain convex polytopes and corona product of a cycle with path in [18], and some classes of plane graphs in [19]. Besides, Irfan et al. [12] conducted a study on this variant of graph labeling on generalized prism graphs. Recently, the authors [3] examined the reflexive edge irregularity strength of the m^t-graphs ($t = 1$) of the cycle graph. For more existing studies on this type of graph labeling, please refer to [1, 11, 13, 14, 20]. The term "zigzag grid graph" is not a single, universally defined graph in graph theory, but rather a description that relates to two main concepts. First concept is zigzag structure, i.e., a data model that generalizes a standard grid or spreadsheet to allow for more irregular and complex connections. Second concept is zigzag product, i.e., a specific operation in graph theory used to construct new graphs with desirable properties (especially in complexity theory). Realizing the importance of properties of zigzag structures, in this paper we investigate edge irregular reflexive labeling in zigzag graphs and establish the precise value of their reflexive edge strength. The findings provide additional evidence for the validity of Conjecture 1.

2. Reflexive edge strength of zigzag graph Z_m^n

Ahmad et al. [2] denoted the zigzag graph as Z_m^n for $m \geq 3$ and $n \geq 2$ (see Figure 1), with m columns and n rows, with vertex set and edge set for Z_m^n are defined as

$$V(Z_m^n) = \{x_{a,b} : 1 \leq a \leq m, 1 \leq b \leq n\}$$

and

$$E(Z_m^n) = \{x_{a,b}x_{a+1,b} : 1 \leq a \leq m-1, 1 \leq b \leq n\} \cup \{x_{a,b}x_{a-1,b+1} : 2 \leq a \leq m, 1 \leq b \leq n-1\},$$

respectively, with $|V(Z_m^n)| = mn$ and $|E(Z_m^n)| = 2mn - 2n - m + 1$.

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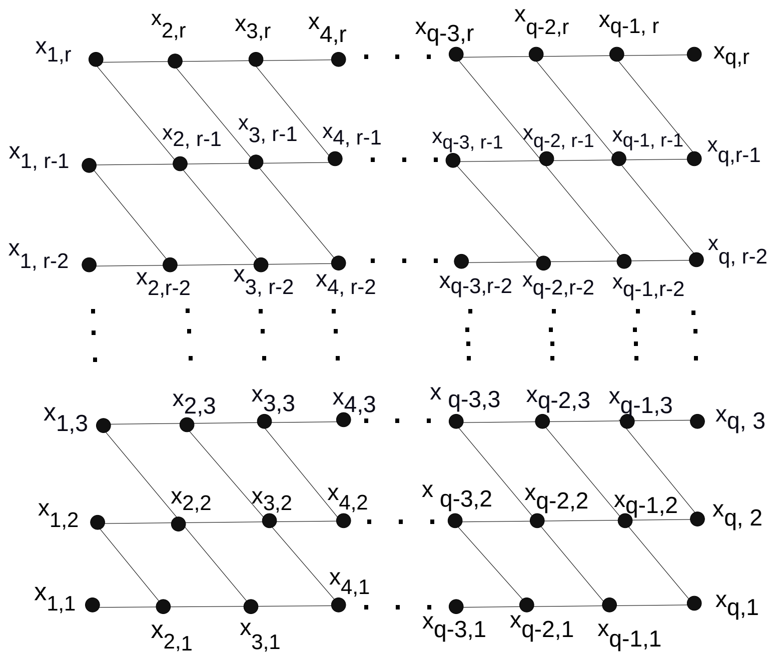

FIGURE 1. Zigzag graph Z_m^n

We establish the reflexive edge strength for Z_m^n in the following result.

Theorem 1. *For Z_m^n where $m \geq 3, n \geq 2$, we have*

$$res(Z_m^n) = \begin{cases} \lceil \frac{2mn-m-2n+1}{3} \rceil + 1, & \text{if} \quad (2mn-m-2n+1) \equiv 2,3 (mod \ \ 6) \\ \lceil \frac{2mn-m-2n+1}{3} \rceil, & \text{if} \quad (2mn-m-2n+1) \not\equiv 2,3 (mod \ \ 6). \end{cases}$$

Proof. We know that Z_m^n has $(2mn-m-2n+1)$ edges. Applying Lemma 1 gives

$$\text{res}(Z_m^n) \geq k = \begin{cases} \lceil \frac{2mn-2n-m+1}{3} \rceil + 1, & \text{if } 2mn-2n-m+1 \equiv 2,3 \pmod 6, \\ \lceil \frac{2mn-2n-m+1}{3} \rceil, & \text{if } 2mn-2n-m+1 \not\equiv 2,3 \pmod 6. \end{cases}$$

Next, we will show that

$$\text{res}(Z_m^n) \leq k = \begin{cases} \lceil \frac{2mn-2n-m+1}{3} \rceil + 1, & \text{if } 2mn-2n-m+1 \equiv 2,3 \pmod 6, \\ \lceil \frac{2mn-2n-m+1}{3} \rceil, & \text{if } 2mn-2n-m+1 \not\equiv 2,3 \pmod 6. \end{cases}$$

We define a total f-labeling of Z_m^n as bellow:

Case 1. When $m \equiv 0 \, (mod \, 6)$,

$$f(x_{a,b}) = \begin{cases} 0, & \text{for } a \in [1,m]; \quad b = 1; \\ l-2, & \text{for } a \in [1,2]; \quad b = 2; \\ l, & \text{for } a \in [3,m]; \quad b = 2; \\ l, & \text{for } a \in [1,m]; \quad b \in [4,n];\ t \equiv 1,2\,(mod\ 3), l\ even; \\ l-1, & \text{for } a \in [1,m]; \quad b \in [3,n];\ t \equiv 0\,(mod\ 3), l\ odd. \end{cases}$$

$$f((x_{a,b})(x_{a+1,b})) = \begin{cases} a, & \text{for } a \in [1,m-1],\ b = 1; \\ 4-a, & \text{for } a \in [1,2],\ b = 2; \\ a-2, & \text{for } a \in [3,m-1],\ b = 2; \\ 2(m-1)(\frac{b-2}{3}) - 2 + a & \text{for } a \in [1,m-1],\ b \in [3,n],\ b \equiv 2\,(mod\ 3) \\ 2(m-1)(\frac{b}{3}) - (\frac{4m}{3} - 2) + a & \text{for } a \in [1,m-1],\ b \in [3,n],\ b \equiv 0\,(mod\ 3) \\ 2(m-1)(\frac{b-1}{3}) - (\frac{2m}{3}) + a & \text{for } a \in [1,m-1],\ b \in [4,n],\ b \equiv 1\,(mod\ 3). \end{cases}$$

$$f((x_{a,b+1})(x_{a+1,b})) = \begin{cases} a+1, & \text{for } a = 1,2,\ b = 1; \\ a-1, & \text{for } a \in [3,m-1],\ b = 1; \\ \frac{m}{3} + 2, & \text{for } a = 1,\ b = 2; \\ \frac{m}{3} + a, & \text{for } a \in [2,m-1],\ b = 2; \\ 2(m-1)(\frac{b}{3}) - (m-1) + a & \text{for } a \in [1,m-1],\ b \in [3,n],\ b \equiv 0(mod\ 3) \\ 2(m-1)(\frac{b-1}{3}) - (\frac{m}{3} + 1) + a & \text{for } a \in [1,m-1],\ b \in [4,n],\ b \equiv 1\,(mod\ 3) \\ 2(m-1)(\frac{b-2}{3}) + (\frac{m}{3} - 1) + a & \text{for } a \in [1,m-1],\ b \in [5,n-1],\ b \equiv 2\,(mod\ 3). \end{cases}$$

Case 2. When $m \equiv 1\,(mod\,6), m \geq 3, n \geq 2$ and for $n[1,m]$,

$$f(x_{a,b}) = \begin{cases} 0, & \text{for } b = 1; \\ l, & \text{for } b \in [2,n], \text{ for } a \in [1,m-1]; \end{cases}$$

$$f((x_{a,b})(x_{a+1,b})) = \begin{cases} a, & \text{for } b = 1; \\ 2(\frac{m-1}{3})b - 4(\frac{m-1}{3}) + a & \text{for } b \in [2,n],\ a \in [1,m-1]; \end{cases}$$

$$f((x_{a,b+1})(x_{a+1,b})) = \begin{cases} a, & \text{for } b = 1; \\ 2(\frac{m-1}{3})b - (m-1) + a & \text{for } b \in [2,n-1]. \end{cases}$$

Case 3. When $m \equiv 2\,(mod\,6)$,

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$$f(x_{a,b}) = \begin{cases} 0, & \text{for } 1 \le a \le m,\ b = 1; \\ l-2, & \text{for } a \in [1,2],\ b = 2; \\ l, & \text{for } a \in [3,m],\ b = 2; \\ l, & \text{for } a \in [1,m],\ b \in [3,n],\ b \equiv 2,3\,(mod\ 3),\ l\ even; \\ l-1, & \text{for } a \in [1,m],\ b \in [4,n],\ b \equiv 1\,(mod\ 3),\ l\ odd; \end{cases}$$

$$f((x_{a,b})(x_{a+1,b})) = \begin{cases} a, & \text{for } a \in [1,m-1],\ b = 1; \\ 4-a, & \text{for } a \in [1,2],\ b = 2; \\ s-2, & \text{for } a \in [3,m-1],\ b = 2; \\ 2(m-1)(\frac{b}{3}) - 4(\frac{m-2}{3}) - 2 + a & \text{for } a \in [1,m-1],\ b \in [3,n],\ b \equiv 0\,(mod\ 3); \\ 2(m-1)(\frac{b-1}{3}) - 2(\frac{m-2}{3}) + a & \text{for } a \in [1,m-1],\ b \in [4,n],\ b \equiv 1\,(mod\ 3); \\ 2(m-1)(\frac{b-2}{3}) - 2 + a & \text{for } a \in [1,m-1],\ b \in [5,n],\ b \equiv 2\,(mod\ 3); \end{cases}$$

$$f((x_{a,b+1})(x_{a+1,b})) = \begin{cases} a+1, & \text{for } a \in [1,2],\ b = 1; \\ a-1, & \text{for } a \in [3,m-1],\ b = 1; \\ \frac{m-2}{3} + 2, & \text{for } a = 1,\ b = 2; \\ \frac{m-2}{3} + a, & \text{for } a \in [2,m-1],\ b = 2; \\ 2(m-1)(\frac{b}{3}) - (m-1) + a & \text{for } a \in [1,m-1],\ b \in [3,n-1],\ b \equiv 0\,(mod\ 3); \\ 2(m-1)(\frac{b-1}{3}) - (\frac{m-2}{3} + 1) + a & \text{for } a \in [1,m-1],\ b \in [4,n-1],\ b \equiv 1\,(mod\ 3); \\ 2(m-1)(\frac{b-2}{3}) + (\frac{m-2}{3} - 1) + a & \text{for } a \in [1,m-1],\ b \in [5,n-1],\ b \equiv 2\,(mod\ 3). \end{cases}$$

Case 4. When $a \in [1,m]$ where $m \equiv 3\,(mod\ 6)$

$$f(x_{a,b}) = \begin{cases} 0, & \text{for } b = 1; \\ l, & \text{for } b \in [2,n]; \end{cases}$$

for $1 \le a \le m-1$,

$$f((x_{a,b})(x_{a+1,b})) = \begin{cases} a, & \text{for } b = 1; \\ 2(m-1)(\frac{b}{3}) - \frac{4m}{3} + a & \text{for } b \in [3,n],\ b \equiv 0\,(mod\ 3); \\ 2(m-1)(\frac{b-2}{3}) + a & \text{for } b \in [2,n],\ b \equiv 2\,(mod\ 3); \\ 2(m-1)(\frac{b-1}{3}) - (\frac{2m}{3} + 2) + a & \text{for } b \in [4,n],\ b \equiv 1\,(mod\ 3); \end{cases}$$

for $1 \le a \le m-1$,

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$$f((x_{a,b+1})(x_{a+1,b})) = \begin{cases} a, & \text{for} \;\; b = 1; \\ 2(m-1)(\frac{b-2}{3}) + (\frac{m}{3} - 1) + a & \text{for} \;\; b \in [2, n-1],\, b \equiv 2\,(mod\ 3); \\ 2(m-1)(\frac{b}{3}) - (m+1) + a & \text{for} \;\; b \in [3, n-1],\, b \equiv 0\,(mod\ 3); \\ 2(m-1)(\frac{b-1}{3}) - (\frac{m}{3} + 1) + a & \text{for} \;\; b \in [4, n-1],\, b \equiv 1\,(mod\ 3). \end{cases}$$

Case 5. When $m \equiv 4\,(mod\ 6)$,

$$f(x_{a,b}) = \begin{cases} 0, & \text{for} \;\; a \in [1, m],\, b = 1; \\ l-2, & \text{for} \;\; a \in [1, 2],\, b = 2; \\ l, & \text{for} \;\; a \in [3, m],\, b = 2; \\ l, & \text{for} \;\; a \in [1, m];\, b \in [1, n]; \end{cases}$$

$$f((x_{a,b})(x_{a+1,b})) = \begin{cases} a, & \text{for} \;\; a \in [1, m-1],\, b = 1; \\ 4-a, & \text{for} \;\; a \in [1, 2],\, b = 2; \\ a-2, & \text{for} \;\; a \in [3, m-1],\, b = 2; \\ (2(\frac{m-4}{3}) + 2)j - (4(\frac{m-4}{3}) + 6) + a & \text{for} \;\; a \in [1, m-1],\, b \in [3, n]; \end{cases}$$

$$f((x_{a,b+1})(x_{a+1,b})) = \begin{cases} a+1, & \text{for} \;\; a \in [1, 2],\, b = 1; \\ a-1, & \text{for} \;\; a \in [3, m-1],\, b = 1; \\ \frac{m-4}{3} + 2, & \text{for} \;\; a = 1,\, b = 2; \\ \frac{m-4}{3} + a, & \text{for} \;\; a \in [2, m-1],\, b = 2; \\ 2(\frac{m-1}{3})b - (m+1) + a & \text{for} \;\; a \in [1, m-1] 1 \le a \le m-1,\, b \in [3, n-1]. \end{cases}$$

Case 6. When $m \equiv 5\,(mod\ 6)$ and for $a \in [1, m]$.

$$f(x_{a,b}) = \begin{cases} 0, & \text{for} \;\; b = 1; \\ l, & \text{for} \;\; b \in [2, n]; \end{cases}$$

for $a \in [1, m-1]$,

$$f((x_{a,b})(x_{a+1,b})) = \begin{cases} a, & \text{for} \;\; b = 1; \\ 2(m-1)(\frac{b-2}{3}) + a & \text{for} \;\; b \in [2, n],\, b \equiv 2\,(mod\ 3); \\ 2(m-1)(\frac{b}{3}) - 4(\frac{m+1}{3}) + a & \text{for} \;\; b \in [3, n],\, b \equiv 0\,(mod\ 3); \\ 2(m-1)(\frac{b-1}{3}) - 2(\frac{m+1}{3}) + a & \text{for} \;\; b \in [4, n],\, b \equiv 1\,(mod\ 3); \end{cases}$$

for $a \in [1, m-1]$,

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$$f((x_{a,b+1})(x_{a+1,b})) = \begin{cases} a, & \text{for } \; b = 1; \\ 2(m-1)(\frac{b-2}{3}) + \frac{m-5}{3} + a & \text{for } \; b \in [2, n-1],\ b \equiv 2\,(mod\ 3); \\ 2(m-1)(\frac{b}{3}) - (m+1) + a & \text{for } \; b \in [3, n-1],\ b \equiv 0\,(mod\ 3); \\ 2(m-1)(\frac{b-1}{3}) - (\frac{m-5}{3} + 2) + a & \text{for } \; b \in [4, n-1],\ b \equiv 1\,(mod\ 3). \end{cases}$$

Calculating the edges weights gives

$$wt((x_{a,b})(x_{a+1,b})) = \left\{ \; 2(m-1)b - 2(m-1) + a, \qquad \text{for } \; a \in [1, m-1],\ b \in [1, n] \right.$$

$$wt((x_{a,b+1})(x_{a+1,b})) = \left\{ \; 2(m-1)b - (m-1) + a, \qquad \text{for } \; a \in [1, m-1],\ b \in [1, n-1]. \right.$$

Hence, any two edges have different weights. Hence, f is an edge irregular reflexive labeling for Z_m^n with $m \geq 3$ and $n \geq 2$. Therefore, we obtain the required result. $\square$

One can check Figure 2 gives the edge irregular reflexive 16-labeling of Z_5^6 and edge weights of the edges in this graph is given in Figure 3.

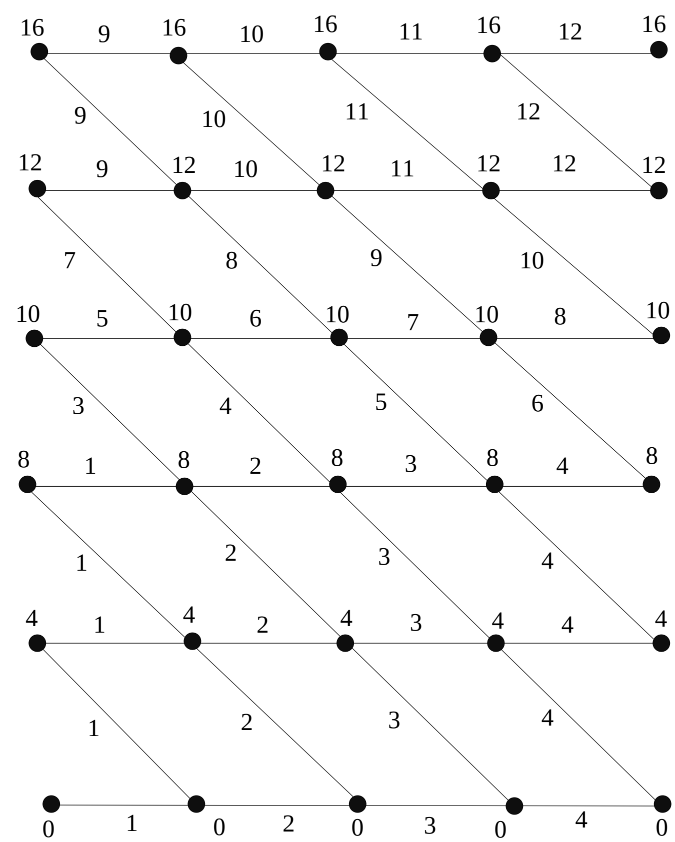

FIGURE 2. The edge irregular reflexive 16-labeling of Z_5^6

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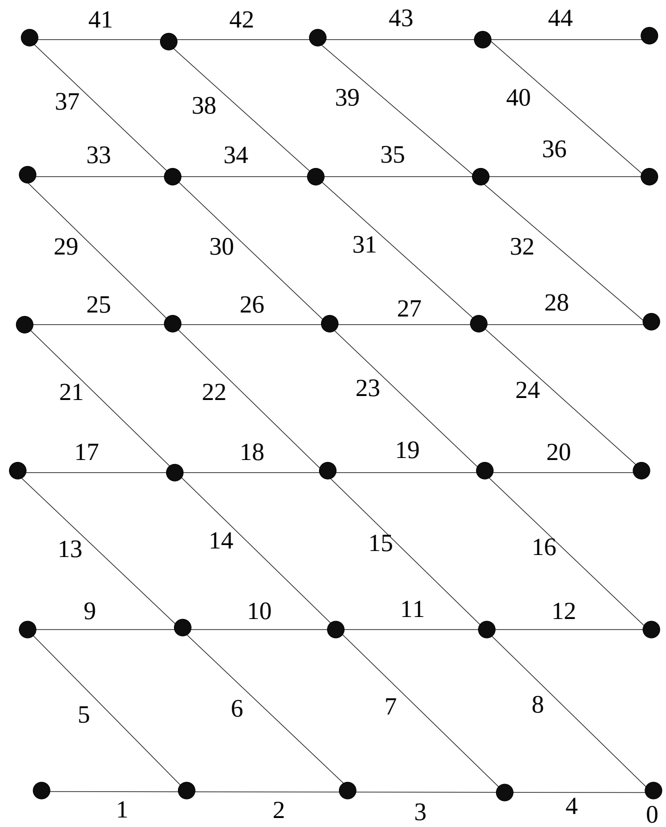

FIGURE 3. The edge weights of the edges in Z_5^6

3. Conclusion

The labeling of graphs has garnered significant attention due to its numerous applications in real life, particularly in networking and network communications. The optimal utilization of infrastructures and achieving connectivity can be obtained through these labelings. Therefore, this paper successfully determined the exact value of the reflexive edge strength of zigzag graph Z_m^n for $m \geq 3$ and $n \geq 2$, which could potentially be further studied and has practical applications in real-life scenarios. This obtained result provided additional support to Conjecture 1.

References

[1] I. H. Agustin, Dafik, M. I. Utoyo, Slamin and M. Venkatachalam, The reflexive edge strength on some almost regular graphs, *Heliyon*, **7** no. 5 (2021) e06991.

[2] A. Ahmad, M. K. Siddiqui and D. Afzal, On the total edge irregularity strength of zigzag graphs, *Australas. J. Combin.*, **54** (2012) 141–149.

https://dx.doi.org/10.22108/toc.2026.146523.2322

[3] M. A. Asif, R. Ismail, A. Razaq, E. H. A. Al-Sabri, M. H. Mateen and S. Ali, An application on edge irregular reflexive labeling for m^t-graph of cycle graph, *AIMS Math.*, **10** no. 1 (2025) 1300–1321.

[4] M. Bača, M. Irfan, J. Ryan, A. Semaničová-Feňovčíková, and D. Tanna, On edge irregular reflexive labelings for the generalized friendship graphs, *Mathematics*, **67** (2017) 2–11.

[5] M. Bača, M. Irfan, J. Ryan, A. Semaničová-Feňovčíková and D. Tanna, Note on edge irregular reflexive labelings of graphs, *AKCE Int. J. Graphs Comb.*, **16** no. 2 (2019) 145–157.

[6] M. Bača, S. Jendrol', M. Miller, and J. Ryan, On irregular total labelings, *Discrete Math.*, **307** (2007) 1378–1388.

[7] M. Basher, On the reflexive edge strength of the circulant graphs, *AIMS Math.*, **6** no. 9 (2021) 9342–9365.

[8] M. Basher, The reflexive edge strength of toroidal fullerene, *AKCE Int. J. Graphs Comb.*, **20** no. 1 (2023) 35–39.

[9] G. Chartrand, M. S. Jacobson, J. Lehel, O. R. Oellermann, S. Ruiz and F. Saba, Irregular networks, *Congr. Numer.*, **64** (1988) 187–192.

[10] J. A. Gallian, A dynamic survey of graph labeling, *Electron. J. Comb.*, **5** 2023, #DS 6.

[11] J. L. G. Guirao, S. Ahmad, M. K. Siddiqui and M. Ibrahim, Edge irregular reflexive labeling for the disjoint union of generalized Petersen graph, *Mathematics*, **6** (2018), 304.

[12] M. Irfan, M. Bača and A. Semaničová-Feňovčíková, On reflexive edge strength of generalized prism graphs, *Electron. J. Graph Theory Appl.*, **10** no. 2 (2022) 415–424.

[13] Y. Ke, M. J. A. Khan, M. Ibrahim and M. K. Siddiqui, On edge irregular reflexive labeling for cartesian product of two graphs, *Eur. Phys. J. Plus*, **136** article number 6 (2021).

[14] M. J. A. Khan, M. Ibrahim and A. Ahmad, On edge irregular reflexive labeling of categorical product of two paths, *Comput. Syst. Sci. Eng.*, **36** no. 3 (2021) 485–492.

[15] J. Ryan, B. Munasinghe, A. Semaničová-Feňovčíková and D. Tanna, Reflexive irregular labelings, submitted.

[16] D. Tanna, J. Ryan and A. Semaničová-Feňovčíková, Edge irregular reflexive labeling of prisms and wheels, *Australas. J. Combin.*, **69** (2017) 394–401.

[17] K. K. Yoong, R. Hasni, M. Irfan, I. Taraweh, A. Ahmad and S.-M. Lee, On the edge irregular reflexive labeling of corona product of graphs with path, *AKCE Int. J. Graphs Comb.*, **18** no. 1 (2021) 53–59.

[18] K. K. Yoong, R. Hasni, G. C. Lau, M. A. Asim and A. Ahmad, Reflexive edge strength of convex polytopes and corona product of cycle with path, *AIMS Math.*, **7** no. 7 (2022) 11784–11800.

[19] K. K. Yoong, R. Hasni, G. C. Lau, and M. Irfan, Edge irregular reflexive labeling for some classes of plane graphs, *Malays. J. Math. Sci.*, **16** no. 1 (2022) 25–36.

[20] X. Zhang, M. Ibrahim, S. A. H. Bokhary and M. K. Siddiqui, Edge irregular reflexive labeling for the disjoint union of gear graphs and prism graphs, *Mathematics*, **6** no. 9 (2018) 142.

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