Adaptive Interval Type-2 Fuzzy Inference System for Facial Expression Recognition

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A B S T R A C T

Fuzzy Inference systems have been successfully applied for different pattern recognition problems. However, in some real-world applications such as facial expression recognition, the uncertainties related to the feature space may be so high that it is hard to model the feature space by fuzzy membership functions. On the other hand, it is believed that type-2 fuzzy sets have high potentials of uncertainty management in the space of features. However, adjusting the parameters of Type 2 membership functions is a difficult task. In this paper, we analyze the effect of incorporating type-2 fuzzy system into facial expression recognition problem. In this regard, two adaptive type-2 fuzzy models are proposed. The first model employs interval type-2 Mamdani fuzzy system for constructing fuzzy face space and uses Genetic Algorithm for optimizing the membership functions parameters. The second model is an interval type-2 Neuro-Fuzzy system which contains interval type-2 fuzzy sets as membership functions. In this case, the gradient descent algorithm is utilized for tuning the parameters of this system. Numerical results demonstrate the superiority of type-2 fuzzy systems with respect to the corresponding type 1 systems and show that the proposed systems can better cope with the uncertainties in facial expression recognition.

1 Introduction

Changes within human face resulted from inner feelings are regarded as facial expressions. Ekman and Friesen\textsuperscript{1} demonstrated that these expressions have the same meaning for all people. They presented six basic expressions which are: anger, happiness, sadness, surprise, disgust, and fear. Facial expression recognition is of high importance in human-machine interactions. After recognizing one’s facial expression and consequently his feeling by machines, a corresponding action to it can be performed. This can improve human-machine interaction.

Although different methods have been proposed for facial expression recognition, in this article, we have employed Fuzzy Inference System (FIS) for this task. The advantage of this system is that its reasoning function is intelligible and is also similar to human logic. Gomathi et al.\textsuperscript{2} proposed a system comprised of several Adaptive Neuro-Fuzzy Inference Systems (ANFISs) and utilized local binary patterns for extracting facial features. Also, they used type-1 fuzzy

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membership functions (T1MFs) in their fuzzification procedures. Ilbeygi and Shah-Hosseini applied a type-1 Mamdani fuzzy system (T1-MFS) to classify facial expressions [3]. Membership functions (MFs) in this system are optimized using Genetic Algorithm. Esau et al. [4] proposed a T1-MFS for facial recognition in fixed images. They considered one output for each face and two MFs for each output. They extracted fuzzy rules manually. They recognized four facial expressions (happiness, sadness, fear and anger) using T1MFs. However, this system is not able to recognize disgust and surprise. Xiang et al. [5] applied discrete Fourier transformation to extract time variations in each pixel and utilized fuzzy c-means for modeling these variations. Khanum et al. [6] presented a facial expression recognition model in colorful images and performed facial expression classification using fuzzy case-based reasoning.

All the methods that are mentioned above are based on type-1 fuzzy systems (T1FSs). T1FS assumes full certainty in fuzzy sets representation. However, in some real world applications such as face expression recognition this is not the case and there exists much more uncertainties that should be considered. Linguistic words in fuzzy rules may have different meanings for various people in facial expression recognition concept. For instance, when calling someone “sad”, the rate of his/her sadness may differ from different people’s point of view. In order to model these uncertainties, type-2 fuzzy sets are used. In these sets, the degree of MFs is a fuzzy type which makes them powerful models for uncertainty management.

Recently, type-2 fuzzy sets have been successfully applied in many real-world applications [1][9]. In the case of expression analysis, Konar et al. [10] used type-2 fuzzy sets for facial expression recognition applications for the first time. Their proposed method is applied when several instances of a facial expression related to the same subject are available. The training data corresponding to each subject are used to estimate a type-1 fuzzy set for each feature. The union of all T1MFs, obtained from various subjects, constructs an interval type-2 fuzzy set. An unknown expression is determined by computing the support of each class for a specific expression. The class having the maximum support is determined as the emotion of the unknown facial expression. Using facial expression instances of the same subject for constructing the T1MFs is a limitation of their method. The more instances are available for each subject as the training set, the more accurate T1MFs are obtained. This requirement is usually not satisfied by facial expression databases. Their system is able to recognize 5 expressions, including anger, fear, disgust, happiness and relaxation. Although their method can capture inter-personal uncertainty level, taking all MFs together can result in a large level of uncertainty which affects the classification accuracy. Besides, no comparison with type-1 fuzzy model is presented.

Halder et al. [11] used general type-2 fuzzy sets to model the fuzzy face space. General type-2 fuzzy set involves primary and secondary MFs. They obtained primary memberships for various subjects using the method of [10]. Hence their method suffer from the same limitations at this stage. Then, they constructed secondary MFs for individual primary membership curves by formulating and solving an optimization problem. However, using general type-2 fuzzy approach results in extensive computational overhead in comparison to the interval approach.

None of the mentioned type-2 fuzzy systems (T2FSs) uses Mamdani or TSK models for inference. Mamdani and TSK are two well-known inference models which have performed successfully on many fuzzy inference problems. In this paper, we aim to analyze the effect of incorporating type-2 fuzzy membership functions into Mamdani and TSK systems to solve the facial expression recognition problem. We provide two interval T2FSs for facial expression recognition. Each system is compared with its corresponding T1FS and the superiority of T2FSs is demonstrated using numerical results.

One of the main stages in designing T2FSs is determining the parameters of fuzzy rules. Recently, several approaches have been proposed for learning and tuning fuzzy systems [12][13]. Here, we use two adaptive type-2 fuzzy models using GA and gradient descent for parameter learning. The first model employs Interval type-2 Mamdani fuzzy system (IT2-MFS) to construct a fuzzy face space. To this end, a T1-MFS is designed and optimized using GA. Then it is extended to the IT2-MFS. The parameters of type-2 membership functions (T2MFs) are also optimized using GA.

The second model is an interval type-2 Neuro-Fuzzy system (IT2-NFS) which is based on a TSK model. Input MFs of the system are interval type-2 fuzzy sets with parameters optimized using gradient descent technique. In order to compare the results, a type-1 Neuro-Fuzzy system (T1-NFS) is also designed using the same rule set as the type-2 system. Parameters of this system are also optimized using the same optimization algorithm. The rest of the paper is organized as follows: Section 2 provides a brief introduction to type-2 fuzzy sets. The provided IT2-MFS and IT2-NFS are described in Sections 3 and 4 respectively. Numerical results are presented in Section 5. In Sections 6 and 7 performance evaluation and conclusions are explained.
2 Preliminaries on Type-2 Fuzzy Sets

The concept of type-2 fuzzy set, introduced as an extension of T1FS, allows us to handle numerical and linguistic uncertainties. The MF of a T2FS is a fuzzy set instead of a crisp value. A T2FS, denoted as $A$, is expressed as [10]:

$$A = \{(x,u), \mu_A(x,u)\} \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \quad (1)$$

Where $x$ and $J_x$ are the primary and the secondary variables respectively. $\mu_A(x,u) \in [0,1]$ is the secondary membership function and its domain, $J_x$, is the primary membership function of $x$, where $J_x \subseteq [0,1] \forall x \in X$. An alternative definition for a T2FS is as follows [10]:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u) \quad J_x \subseteq [0,1] \quad (2)$$

Where $\int \int$ denotes the union over all possible values of $x$ and $u$. An interval T2FS is a special case of T2FS in which the amplitude of the secondary membership functions, called secondary grades, equals unity [16]. An interval T2FS is described as:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x,u) \quad J_x \subseteq [0,1] \quad (3)$$

The uncertainty about T2MF $\tilde{A}$ is shown by the union of all primary membership grades, called the footprint of uncertainty (FOU), i.e. $FOU(\tilde{A}) = \bigcup_{x \in X} J_x$ [10]. The FOU is bounded by two T1MFs: upper MF (UMF) and lower MF (LMF). UMF and LMF, denoted by $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{A}}(x)$, are two subsets that have the maximum and minimum membership grades of the footprint of uncertainty respectively [17]. An interval T2FS can be completely described by its LMF and UMF.

3 IT2-MFS for Facial Expression Recognition

As said before, one method to determine the parameters of type-2 fuzzy sets is based on type-1 parameters. We learn an interval type-2 fuzzy membership function based on T1FS. At first step, we present a T1FS for recognizing different expressions and then we extend it to the interval type-2 fuzzy inference system and optimize the parameters of the MFs using GA.

3.1 T1-MFS

In our T1-MFS, the parameters are initialized according to intuitive expert’s knowledge. Fuzzy rules are then extracted experimentally based on these MFs. Finally, T1MF parameters are optimized using GA.

The proposed T1-MFS has nine input features, which are extracted using the procedure described in section VI, and twelve outputs related to six expressions. For each expression, two outputs, i.e. “strong_Emotion” and “weak_Emotion” are considered. This can help for better accuracy because we can separate high intensity expressions from normal intensity expressions. If either one of these outputs has the highest support we select the related expression as the final output of the system. Each output is modeled using two Gaussian MFs named as “Low” and “High”. Hence, the defuzzified output is a vector comprised of twelve crisp values which are the centers of the related output fuzzy sets.

If the numbers of MFs for inputs are less than enough, fuzzy model and system accuracy will diminish. Moreover, since this system uses GA, choosing several MFs would result in aggregating system parameters, leading to an increased chromosome length. Therefore, to accomplish the fuzzification process, for each feature three MFs is considered. These functions are Low, Medium and High and of Gaussian type. All features are normalized and then scaled into [0, 1] domain.

Fuzzy rules are extracted experimentally based on the defined MFs. An example of these rules which corresponds to “strong surprise” expression is as follows:

Rule: If $(mouth_width \text{ is Low}) \text{ AND } (mouth_opening \text{ is Low}) \text{ AND } (inner_left_eyebrow_height \text{ is High}) \text{ AND } (inner_right_eyebrow_height \text{ is High}) \text{ AND } (middle_left_eyebrow_height \text{ is High}) \text{ AND } (middle_right_eyebrow_height \text{ is High}) \text{ AND } (eye_to_mouth_corners_mean \text{ is Medium}) \text{ AND } (left_eye_opening \text{ is High}) \text{ AND } (right_eye_opening \text{ is High})$, Then $(weak_angry \text{ is Low}) \text{ AND } (strong_angry \text{ is Low}) \text{ AND } (weak_happy \text{ is Low}) \text{ AND } (strong_happy \text{ is Low}) \text{ AND } (weak_sad \text{ is Low}) \text{ AND } (strong_sad \text{ is Low}) \text{ AND } (weak_fear \text{ is Low}) \text{ AND } (strong_fear \text{ is Low}) \text{ AND } (weak_disgust \text{ is Low}) \text{ AND } (strong_disgust \text{ is Low}) \text{ AND } (weak_surprise \text{ is Low}) \text{ AND } (strong_surprise \text{ is High})$

One of the main factors in the accuracy of a fuzzy based system is the parameters related to the MFs which are tuned using GA in this paper. Since each Gaussian function has two variables, the number of parameters related to the input and the output MFs are $2^3 \times 9 = 54$ and $2^2 \times 12 = 48$ respectively. Hence, the length of the chromosome will be $54 + 48 = 102$.

In order to prepare target vector which would be compared to the system output, we assign the “weak
emotion” or “strong emotion” label to each training data manually. Hence, the related value will be set to one in the target vector, while other eleven values are set to zero. Having calculated the optimized parameters of the T1FLS, we can extend it to T2FLS.

3.2 IT2-MFS

After constructing T1-MFS and optimizing the parameters of the MFs using GA, we generalized it to IT2-MFS. The overall structure of IT2-MFS is depicted in Figure 1.

One of the main tasks in designing T2FLS is to model FOU. In this paper, interval T2MFs are generated using a primary MF with a fixed standard deviation but uncertain mean [17]. In other words, FOU is comprised of numerous Gaussian functions with uncertain centers as depicted in Figure 2. Having designed a Gaussian T1MF, \( N(\sigma, c, x) \), if a constant \( \delta \) is added to or subtracted from \( c \), two centers called \( c_1 \) and \( c_2 \) would be obtained. The upper MF of the obtained T2MF is defined by:

\[
\mu_U = \begin{cases} 
N(c_1, \sigma, x), & x < c_1 \\
1, & c_1 < x < c_2 \\
N(c_2, \sigma, x), & c_2 < x 
\end{cases} \quad (4)
\]

While the lower MF is:

\[
\mu_L = \begin{cases} 
N(c_2, \sigma, x), & x < (c_1 + c_2)/2 \\
N(c_1, \sigma, x), & (c_1 + c_2)/2 \leq x 
\end{cases} \quad (5)
\]

As stated before, the parameter \( \delta \) is optimized using GA. The number of MFs in the designed T1FLS is \( 3^9 + 2^12 = 51 \). Hence, there are 51 parameters to be optimized. More details about genetic algorithm are presented in Section 5. Fuzzy rules extracted from the previous model are used in this system as well, with the difference that the T1FSs are replaced with the corresponding T2FSs in T2 fuzzy system. The summary of the procedure of designing T2 Mamdani fuzzy system is presented in Algorithm 1.

It should be noted that [15] also uses GA to learn the MF parameters of a T2-MFS. The main differences between our work and [15] are as following: we optimize T1 parameters before optimizing T2 parameters. Better initialization of T2 parameters and the two-stage optimization leads to better performance. Also, unlike [15], in our method the chromosome length is independent from the number of rules. So the complexity of GA does not increase as the number of rules increases. The other difference is that we consider two subclasses (weak and strong) for each class in our rule set to define more accurate rules and achieve more accuracy.

4 IT2-NFS for Facial Expression Recognition

For the next fuzzy model, we have performed expression recognition using type-1 and type-2 Neuro-Fuzzy
systems. In both cases, the parameters of system are optimized using the gradient descent algorithm. In the following sections, at first, type-1 system is described briefly and then type-2 system is presented in detail.

4.1 T1-NFS

In our designed T1FS each input is described using three MFs, i.e. Low, Medium and High. Two functions Low and High, which are both constant (0 or 1), are defined for outputs of this TSK system. The rule base of the system consists of $R$ rules, where the $r$th rule is as follows:

Rule $r$: if $x_1$ is $A_{r}^{1}$ and ... and $x_9$ is $A_{r}^{9}$, ... then $y_1 = z_{r}^{1}$ and ... and $y_6 = z_{r}^{6}$ \hspace{1cm} (6)

Where $r = 1, ..., R$ is the rule number and $x_k$ and $y_j$ ($k = 1, ..., 9 ; j = 1, ..., 6$) are the inputs and the outputs in rule $r$: $A_{r}^{k}$ is the Gaussian MF corresponding to $k$th input of rule $r$. $z_{r}^{j}$ are constant numbers 0 or 1 for each emotion class $j$ in the $r$th rule. The output function for an emotion class is 1 if the emotion is activated, and 0 otherwise. For an input vector $x = (x_1, ..., x_9)$, the fuzzification is performed as follows:

\[
\mu_{k,m} = \exp\left(-\frac{1}{2}\frac{(x_k - c_{k,m})^2}{\sigma_{k,m}^2}\right) \hspace{1cm} (7)
\]

Where $k = 1, ..., 9$ is the input index and $m = 1, 2, 3$ is the MF number. The firing strength of the $r$th rule is defined as:

\[
f_r = \mu_{A_r^{1}}(x_1) \times \ldots \times \mu_{A_r^{9}}(x_9) \hspace{1cm} (8)
\]

Where $A_r^{k}$ is the MF of input $x_k$ in rule $r$, and $\mu_{A_r^{k}}(x_k)$ is the membership degree of this input. The output vector of the system, which contains six crisp values, is calculated as follows:

\[
U = \frac{\sum_{r=1}^{R} f_r Z_r}{\sum_{r=1}^{R} f_r} \hspace{1cm} (9)
\]

Where $f_r$ and $Z_r$ are the firing strength and the output vector of rule $r$ respectively. In order to select appropriate parameters for input MFs, the steepest descent algorithm is used. Each Gaussian function has two variables, so $2^6 = 64$ parameters are optimized. The error of the system is calculated using the following formula:

\[
E = \frac{1}{2} \sum_{p=1}^{N} \sum_{i=1}^{6} (u_{i,p}^d - u_{i,p})^2 \hspace{1cm} (10)
\]

Where $u_{i,p}$ and $u_{i,p}^d$ are the calculated output of the system and the desired value of $i$th output for $p$th training data, respectively. Accordingly, the update formulas for parameters $c_{k,m}$ and $\sigma_{k,m}$ are given by the following equations:

\[
c_{k,m}(t+1) = c_{k,m}(t) - \gamma \frac{\partial E}{\partial c_{k,m}} \hspace{1cm} (11)
\]

\[
\sigma_{k,m}(t+1) = \sigma_{k,m}(t) - \gamma \frac{\partial E}{\partial \sigma_{k,m}} \hspace{1cm} (12)
\]

Where $\gamma$ is the learning rate. Derivation of these formulas is given in Appendix A.

4.2 IT2-NFS

In this section, an interval type-2 neuro-fuzzy inference system is presented. This neuro-fuzzy system contains five layers, as shown in Figure 3. The inputs of this network are the normalized features extracted from the database images. This system has six outputs which determine the support for each emotion class.

The rules used in this model are similar to those used in type-1 model except that the input values in each rule are fuzzified using interval type-2 fuzzy sets. The output functions of these rules are the same as the corresponding type-1 model. IT2-NFS is modeled with a rule base of $R$ rules, each having 9 antecedents. The $r$th rule can be expressed as follows:
Rule $r$: if $x_1$ is $\tilde{A}_1^r$ and ... and $x_9$ is $\tilde{A}_9^r$, ... then $y_1 = z_1^r$ and ... and $y_6 = z_6^r$  

All the variables are the same as the expression (6) except that $\tilde{A}_k^r (k = 1, ..., 9; r = 1, ..., R)$ is the interval type-2 fuzzy set corresponding to $k$th input of rule $r$. The layers of the system are described below:

Layer 1: In this layer, inputs of the system are fuzzified. Three interval type-2 sets, i.e., Low, Medium and High are assumed for each input. These sets are all type-2 Gaussian sets with uncertain mean.

Layer 2: This layer calculates firing strength of type-2 fuzzy rules which are intervals. The left and right limits of these intervals are calculated as follows:

\[
\tilde{f}_r(x) = \mu_{\tilde{A}_1^r}(x_1) \times \cdots \times \mu_{\tilde{A}_9^r}(x_9) \tag{14}
\]

\[
\tilde{f}_r(x) = \tilde{\pi}_{\tilde{A}_1^r}(x_1) \times \cdots \times \tilde{\pi}_{\tilde{A}_9^r}(x_9) \tag{15}
\]

Where $\tilde{\pi}_{\tilde{A}_k^r}(x_k)$ and $\mu_{\tilde{A}_k^r}(x_k)$ are lower and upper bounds of primary membership degree of input $x_k$ in the $r$th rule.

Layer 3: Having calculated the left and right limit of firing strength of each rule, two vectors $U_l$ and $U_r$ are calculated as follows in this layer:

\[
U_l = \frac{\sum_{r=1}^{R} \tilde{f}_r Z^*}{\sum_{r=1}^{R} \tilde{f}_r} \tag{16}
\]

\[
U_r = \frac{R \sum_{r=1}^{R} \tilde{f}_r Z^*}{\sum_{r=1}^{R} \tilde{f}_r} \tag{17}
\]

Where $Z^*$ is the output vector of rule $r$.

Layer 4: The final output of the system is obtained as follows:

\[
U = (q \times U_l) + ((1 - q) \times U_r) \tag{18}
\]

This form of inference replaces type-reduction process in the method presented by [18]. The parameter $q$ is a design factor which indicates the contribution of upper and lower values in the final output.

Layer 5: The emotion class which has the maximum support offered among all 6 classes is selected as the emotion class of the input facial expression.

Similar to T1-NFS, parameters of this system is optimized using the steepest descent algorithm. This system has 27 input fuzzy sets and each set has three parameters to be optimized, i.e. two centers and one standard deviation, so the number of optimization parameters is 81. The training data used to train this system is similar to type-1 system and the error function is defined the same as Equation (19).

Parameters of this system are optimized using the following equations while their derivations are presented in appendix B.

\[
c_1,k,m(t + 1) = c_1,k,m(t) - \gamma \frac{\partial E}{\partial c_1,k,m} \tag{19}
\]
\[ e_{2,k,m}(t + 1) = e_{2,k,m}(t) - \gamma \frac{\partial E}{\partial e_{2,k,m}} \quad (20) \]

\[ \sigma_{k,m}(t + 1) = \sigma_{k,m}(t) - \gamma \frac{\partial E}{\partial \sigma_{k,m}} \quad (21) \]

5 Experimental Results

In this section, we present the experimental results of facial expression recognition using type-2 fuzzy inference models introduced in two previous sections, and compare them with their type-1 counterparts. First, feature extraction procedure and an introduction to a well-known database are presented and then the experimental results are described.

5.1 Feature Extraction and Database

Many papers presume the face acquisition as a provided step and expression recognition is applied to images showing the face part [3, 19]. Likewise, in this paper, it is assumed that the part of the image including the face is extracted previously.

After detecting a face in a given picture, facial feature extraction is performed in order to classify different facial expressions. In this paper geometrical features are selected as the input of our systems. Therefore, 17 points similar to MPEG-4 standard are used [20], which are shown in Figure 4. Euclidian distances between these points form nine features. These features are: mouth width, mouth opening, height of inner eyebrows, height of middle of eyebrows, eye opening, and the means of distances between inner eye corners and outer lip corners. Table 1 represents the definitions of these features according to the points in Figure 4.

Different faces have different sizes and facial features of each person depend on the size of his face, so we normalize the features using facial animation parameter units (FAPU) of MPEG-4 standard. Each FAPU is extracted from a neutral face. These units are the distance between eyes (ES0), the eye opening (IRISD0), the distance between the middle of two eyes and nose (ENS0), the distance between nose and the middle of lips, and mouth width of a neutral face. In order to normalize facial features of a face representing an emotion, first a face with neutral expression is selected as the reference image. FAPUs are then extracted from this image. Finally, normalized features are obtained by dividing each feature to the corresponding FAPU.

Our experiments are performed on JAFFE [21] and Cohn-Kanade [22] databases. JAFFE database contains 213 images of ten Japanese females, each of them expressing six expressions anger, happiness, sadness, fear, disgust and surprise. They also express a neutral facial expression. Each female has two to four instances of expressions for a given emotion. Cohn-Kanade database includes 486 sequences of images from frontal face expressions of 97 people. Sequences of images that display the motion of face, start from neutral face and end to the facial expression. In this database each person represents one or more emotions. Also many of the sequences do not represent one facial expression completely. Table 2 shows the number of images for each expression used in our experiments. Some sample images from JAFFE and Cohn-Kanade face databases are given in Figure 5 and Figure 6 respectively.

For JAFFE database, we extracted the coordinates of 17 points on each face image manually in Mat- lab environment and extracted the corresponding features by computing the defined distances between these points. For Cohn-Kanade, we use the coordinates of points that are marked manually by [23]. They have tracked 59 marked points on each face and their coordinates have been saved in a text file. We used 17 points of these in our experiments.

5.2 Expression Recognition Results

In this section, we present the experimental results obtained from applying Mamdani and Neuro fuzzy inference systems. It should be noted that for some parts of our implementations, we utilize the functions in IT2FLT toolbox of [24].
Table 1. Definition of Facial Features

<table>
<thead>
<tr>
<th>Feature</th>
<th>FAPU</th>
<th>Feature Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of right eyebrows</td>
<td>ENS0</td>
<td>D1 = d(1,4)</td>
</tr>
<tr>
<td>Tight eye opening</td>
<td>IRISD0</td>
<td>D2 = d(2,4)</td>
</tr>
<tr>
<td>Height of right eye inner corner</td>
<td>ENS0</td>
<td>D3 = d(5,6)</td>
</tr>
<tr>
<td>Height of left eyebrows</td>
<td>ENS0</td>
<td>D4 = d(7,10)</td>
</tr>
<tr>
<td>Left eye opening</td>
<td>IRISD0</td>
<td>D5 = d(8,10)</td>
</tr>
<tr>
<td>Height of left eye inner corner</td>
<td>ENS0</td>
<td>D6 = d(11,12)</td>
</tr>
<tr>
<td>Mouth opening</td>
<td>MNS0</td>
<td>D7 = d(14,15)</td>
</tr>
<tr>
<td>Mouth width</td>
<td>MW0</td>
<td>D8 = d(16,17)</td>
</tr>
<tr>
<td>Mean of distance between eyes corners and mouth corners</td>
<td>ENS0</td>
<td>(D9 + D10)/2 = (d(6,16) + d(12,7))/2</td>
</tr>
</tbody>
</table>

Table 2. Number of Used Images for Each Expression in JAFFE and Cohn-Kanade Face Database

<table>
<thead>
<tr>
<th></th>
<th>Surprise</th>
<th>Disgust</th>
<th>Fear</th>
<th>Sadness</th>
<th>Happiness</th>
<th>Anger</th>
</tr>
</thead>
<tbody>
<tr>
<td>JAFFE</td>
<td>30</td>
<td>29</td>
<td>32</td>
<td>31</td>
<td>31</td>
<td>30</td>
</tr>
<tr>
<td>Cohn-Kanade</td>
<td>83</td>
<td>31</td>
<td>44</td>
<td>63</td>
<td>90</td>
<td>25</td>
</tr>
</tbody>
</table>

*Mamdani systems results:* In this section, T2-MFS is compared with its Type-1 counterpart. For T1-MFS, 102 parameters (including mean and variance of Gaussian input and output MFs) are optimized using GA. Optimization is implemented using OPTIMTOOL in Matlab.

A 5-fold cross validation technique is used for optimization. For cross validation, we randomly divide the dataset to five equal size partitions. One partition is used as test set, one as validation set and three others are used as train set. Having optimized the parameters of T1-MFS, we calculate the optimum values for 51 parameters of IT2-MFS, i.e. δ values related to each MFs. Optimization for both algorithms are performed using GA with the parameters depicted in Table 3. An initial population of length 80 with real-value representation is used. The accuracy of the classification algorithm using each parameter set is considered as the fitness value for a given generation. In order to compare the IT2-MFS with its type-1 counterpart, we use a 5-fold cross validation technique with the same data partitions as T1-MFS. Table 4 shows the results of the experiments on JAFFE database. For a better comparison of T1-MFS and IT2-MFS, we present the results as a confusion matrix where each column of the matrix shows the system output while each row shows the desired output. In this table, T1 and T2 denote the results of T1-MFS and IT2-MFS respectively.

Table 3. GA Parameters for Optimizing T1-MFS and IT2-MFS

<table>
<thead>
<tr>
<th>Options</th>
<th>T1-MFS</th>
<th>IT2-MFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population type</td>
<td>Double vector</td>
<td>Double vector</td>
</tr>
<tr>
<td>Population size</td>
<td>150</td>
<td>80</td>
</tr>
<tr>
<td>Fitness scaling</td>
<td>Rank</td>
<td>Rank</td>
</tr>
<tr>
<td>Selection</td>
<td>Tournament</td>
<td>Tournament</td>
</tr>
<tr>
<td>Crossover</td>
<td>Two point</td>
<td>Scattered</td>
</tr>
<tr>
<td>Mutation</td>
<td>Uniform</td>
<td>Uniform</td>
</tr>
</tbody>
</table>

As Table 4 shows, for almost all expressions, IT2-MFS has better performance with respect to T1-MFS on JAFFE. The increase in recognition rate of “disgust” expression is more than other expressions while in the case of “surprise” expression, there is not any improvement although for this expression the highest accuracy is achieved. This is due to the high difference in features of surprise expression in comparison with those of other expressions in this dataset which causes no need to add extra uncertainties to the membership functions.

Table 5 reports the results of the T1-MFS and IT2-MFS on Cohen-Kanade face database. As can be seen in this table, again type-2 fuzzy inference system improves the recognition results, and this improvement
Table 4. Confusion Matrix for T1-MFS and IT2-MFS on JAFFE Database

<table>
<thead>
<tr>
<th>Desired output</th>
<th>System output (%)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>anger</td>
<td>T1</td>
<td>T2</td>
<td>T1</td>
<td>T2</td>
<td>T1</td>
</tr>
<tr>
<td>anger</td>
<td>86.67</td>
<td>93.33</td>
<td>3.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>happiness</td>
<td>0</td>
<td>0</td>
<td>90.32</td>
<td>93.55</td>
<td>6.54</td>
</tr>
<tr>
<td></td>
<td>sadness</td>
<td>9.68</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>80.65</td>
</tr>
<tr>
<td></td>
<td>fear</td>
<td>0</td>
<td>0</td>
<td>6.25</td>
<td>6.54</td>
<td>9.37</td>
</tr>
<tr>
<td></td>
<td>disgust</td>
<td>6.9</td>
<td>3.45</td>
<td>3.45</td>
<td>0</td>
<td>6.9</td>
</tr>
<tr>
<td></td>
<td>surprise</td>
<td>0</td>
<td>0</td>
<td>3.33</td>
<td>3.33</td>
<td>0</td>
</tr>
<tr>
<td>total rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T1</td>
<td>85.79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>90.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5. Confusion Matrix for T1-MFS and IT2-MFS on Cohen-Kanade Database

<table>
<thead>
<tr>
<th>Desired output</th>
<th>System output (%)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>anger</td>
<td>T1</td>
<td>T2</td>
<td>T1</td>
<td>T2</td>
<td>T1</td>
</tr>
<tr>
<td>anger</td>
<td>76</td>
<td>80</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>happiness</td>
<td>0</td>
<td>0</td>
<td>83.33</td>
<td>88.89</td>
<td>3.33</td>
</tr>
<tr>
<td></td>
<td>sadness</td>
<td>3.17</td>
<td>3.17</td>
<td>3.17</td>
<td>3.17</td>
<td>80.95</td>
</tr>
<tr>
<td></td>
<td>fear</td>
<td>4.55</td>
<td>4.55</td>
<td>2.27</td>
<td>0</td>
<td>6.82</td>
</tr>
<tr>
<td></td>
<td>disgust</td>
<td>0</td>
<td>0</td>
<td>12.9</td>
<td>12.9</td>
<td>3.23</td>
</tr>
<tr>
<td></td>
<td>surprise</td>
<td>0</td>
<td>0</td>
<td>2.41</td>
<td>2.41</td>
<td>2.41</td>
</tr>
<tr>
<td>total rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T1</td>
<td>82.74</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>86.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

can be seen in all expressions.

**Neuro-Fuzzy systems results**: As stated before, parameters of both type-1 and type-2 fuzzy systems are optimized using the steepest descent algorithm. A 5-fold cross validation technique, with the same data partitions as for Mamdani system, is used for this purpose. One of the major problems in the optimization algorithm is convergence; a high learning rate may prevent convergence while a low learning rate slows down the convergence process. In order to solve this problem, adaptive learning rate is utilized. Table 6 shows the confusion matrices resulted from using type-1 and interval type-2 Neuro-Fuzzy inference systems for Jaffe dataset. As can be seen from this table, except happiness and surprise expressions, improvement has been achieved for other expressions.

Table 6. Confusion Matrix for T1-MFS and IT2-MFS on JAFFE Database

<table>
<thead>
<tr>
<th>Desired output</th>
<th>System output (%)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>anger</td>
<td>T1</td>
<td>T2</td>
<td>T1</td>
<td>T2</td>
<td>T1</td>
</tr>
<tr>
<td>anger</td>
<td>86.67</td>
<td>93.33</td>
<td>3.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>happiness</td>
<td>0</td>
<td>0</td>
<td>90.32</td>
<td>93.55</td>
<td>6.54</td>
</tr>
<tr>
<td></td>
<td>sadness</td>
<td>9.68</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>80.65</td>
</tr>
<tr>
<td></td>
<td>fear</td>
<td>0</td>
<td>0</td>
<td>6.25</td>
<td>6.54</td>
<td>9.37</td>
</tr>
<tr>
<td></td>
<td>disgust</td>
<td>6.9</td>
<td>3.45</td>
<td>3.45</td>
<td>0</td>
<td>6.9</td>
</tr>
<tr>
<td></td>
<td>surprise</td>
<td>0</td>
<td>0</td>
<td>3.33</td>
<td>3.33</td>
<td>0</td>
</tr>
<tr>
<td>total rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T1</td>
<td>85.79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>90.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6 Performance Evaluation

According to the obtained results, it seems that type-2 systems outperform type-1 systems in both Mamdani and Neuro-Fuzzy models. To determine the validity of this comparison, the statistical test called Wilcoxon signed-rank test is performed to analyze the relative performance of type-2 systems over type-1 counterparts. The Wilcoxon signed-rank test is a non-parametric test that compares two paired groups. The test calculates the difference between each set of pairs and analyzes these differences. The Wilcoxon signed-rank test can be used as an alternative to the t-test to produce a null hypothesis when the population data does not follow a normal distribution. The null hypothesis is that the median difference between pairs of observations is zero. Table 7 shows the result of comparing type-1 and type-2 systems by Wilcoxon test. In
Table 6. Confusion Matrix for T1-NFS and IT2-NFS on JAFFE Database

<table>
<thead>
<tr>
<th>Desired output</th>
<th>System output (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>anger</td>
</tr>
<tr>
<td>anger</td>
<td>86.67</td>
</tr>
<tr>
<td>happiness</td>
<td>0</td>
</tr>
<tr>
<td>sadness</td>
<td>6.45</td>
</tr>
<tr>
<td>fear</td>
<td>0</td>
</tr>
<tr>
<td>disgust</td>
<td>10.34</td>
</tr>
<tr>
<td>surprise</td>
<td>0</td>
</tr>
<tr>
<td>total rate</td>
<td>87.98</td>
</tr>
</tbody>
</table>

Table 7. Wilcoxon Signed-Rank Test Results on Mamdani and Neuro-Fuzzy Systems

<table>
<thead>
<tr>
<th>Database</th>
<th>Type 1</th>
<th>Type 2</th>
<th>Null Hypothesis</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mamdani system</td>
<td>85.79%</td>
<td>90.71%</td>
<td>reject</td>
<td>0.043</td>
</tr>
<tr>
<td>Neuro-Fuzzy system</td>
<td>87.98%</td>
<td>91.08%</td>
<td>reject</td>
<td>0.042</td>
</tr>
</tbody>
</table>

Table 8. Classification Accuracies of Different Methods for JAFFE Dataset

<table>
<thead>
<tr>
<th>Method</th>
<th>Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-MFS</td>
<td>85.79</td>
</tr>
<tr>
<td>IT2-MFS</td>
<td>90.71</td>
</tr>
<tr>
<td>T1-NFS</td>
<td>87.98</td>
</tr>
<tr>
<td>IT2-NFS</td>
<td>91.8</td>
</tr>
<tr>
<td>SVM</td>
<td>84.1</td>
</tr>
<tr>
<td>MLP</td>
<td>84.15</td>
</tr>
</tbody>
</table>

After showing the superiority of type-2 systems to type-1 systems, we compare the obtained results with two well-known classifying methods and also the interval type-2 fuzzy method proposed by [10] [11]. The two classifiers used for comparison are SVM and multi-layer perceptron (MLP). In both cases, a 5-fold cross validation technique, with the same data partitions as our type-2 inference systems, is used for reporting results. The classification accuracy of the mentioned methods on Jaffe database is shown in Table 8. The results show that the proposed type-2 models outperform these two classifiers.

To the best of our knowledge, the method of [10] [11] is the only interval T2FS which has been proposed to solve the facial expression recognition problem. As said before, they need several instances of facial expressions from the same subjects to construct fuzzy face space. They consider 20 subjects, each having 10 instances of a given emotion from Indian women database (Jadavpur University). This database is not publicly available; hence, in order to compare the methods, we implemented their method using JAFFE database in which for each of 10 subjects, two to four instances are available for each emotion. Using 5-fold cross validation scheme, we design the fuzzy space considering 8 subjects and the facial expressions re-

this test, null hypothesis means equal performance of two systems on the same database. Having adjusted the significance level =0.05, if the computed p-value is under 0.05, this hypothesis is rejected. Rejection means that there exists a meaningful difference between the performances of two systems. Hence, according to the obtained accuracies, we can conclude the superiority of type-2 system to type-1 system in both Mamdani and Neuro-Fuzzy methods.

Learning and tuning parameters is performed offline in IT2-MFS and IT2-NFS. One advantage of Neuro-Fuzzy systems over Mamdani ones is their high speed in calculating the output due to the simplicity of the defuzzification process. However, it suffers from the problem of getting caught in local minimums.

After showing the superiority of type-2 systems to type-1 systems, we compare the obtained results with two well-known classifying methods and also the interval type-2 fuzzy method proposed by [10] [11]. The two classifiers used for comparison are SVM and multi-layer perceptron (MLP). In both cases, a 5-fold cross validation technique, with the same data partitions as our type-2 inference systems, is used for reporting results. The classification accuracy of the mentioned methods on Jaffe database is shown in Table 8. The results show that the proposed type-2 models outperform these two classifiers.

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lated to remaining two subjects are used to validate their proposed fuzzy classifier. The resulted accuracies are depicted in Table 3. The poor classification accuracy for interval type-2 method of [11] shows how the number of subjects and their related expression instances can intensely affect the results.

7 Conclusion

This paper utilized interval type-2 fuzzy logic based approach to handle high levels of uncertainty in facial expression recognition problem. Two adaptive interval type-2 fuzzy systems were presented in a way that the parameters of FOU were learned based on type-1 membership functions. The first model was a GA based interval type-2 Mamdani fuzzy model while the other model was a gradient descent based interval type-2 neuro fuzzy system. These two models were statistically compared with their type-1 counterparts and the obtained results demonstrated that type-2 inference methods can better handle the large uncertainties in facial expression recognition problem.

References


A Derivation of BP Algorithm Formulas for T1-NFS

Derivatives used in Equations (11) and (12) are calculated as follows:

\[
\frac{\partial E}{\partial c_{k,m}} = \sum_{p=1}^{N} \sum_{i=1}^{6} \sum_{r=1}^{R} \frac{\partial E}{\partial u_{i,p}} \frac{\partial u_{i,p}}{\partial f_{p,r}} \frac{\partial f_{p,r}}{\partial f_{p,r}} \frac{\partial f_{p,r}}{\partial c_{k,m}},
\]

(A.1)

\[
\frac{\partial E}{\partial \sigma_{k,m}} = \sum_{p=1}^{N} \sum_{i=1}^{6} \sum_{r=1}^{R} \frac{\partial E}{\partial u_{i,p}} \frac{\partial u_{i,p}}{\partial f_{p,r}} \frac{\partial f_{p,r}}{\partial f_{p,r}} \frac{\partial f_{p,r}}{\partial \sigma_{k,m}},
\]

(A.2)

Where k, m, p, i and r indicate the numbers related to input index, MF, training data, system output and fuzzy rule, respectively. Using Equations (8)-10 we have:

\[
\frac{\partial E}{\partial u_{i,p}} = (u_{i,p} - u_{d,i,p}),
\]

(A.3)

\[
\frac{\partial E}{\partial u_{i,p}} = \frac{z_{p,r} - u_{p,r}}{\sum_{i=1}^{N} z_{i,t}},
\]

(A.4)

\[
\frac{\partial f_{p,r}}{\partial \mu_{k,m,i}} = \frac{9}{i=1 \neq k} \mu_{p,i,m},
\]

(A.5)

And according to Equation (7):

\[
\frac{\partial \mu_{p,k,m}}{\partial c_{k,m}} = \frac{x_{p,k} - c_{k,m}}{\sigma_{k,m}^2} \times \exp\left(-\frac{1}{2} \frac{(x_{p,k} - c_{k,m})^2}{\sigma_{k,m}^2}\right),
\]

(A.6)

\[
\frac{\partial \mu_{p,k,m}}{\partial \sigma_{k,m}} = \frac{(x_{p,k} - c_{k,m})^2}{\sigma_{k,m}^2} \times \exp\left(-\frac{1}{2} \frac{(x_{p,k} - c_{k,m})^2}{\sigma_{k,m}^2}\right),
\]

(A.7)

By substituting these derivations in Equations (11) and (12) the parameters of MFs are updated.

B Derivation of BP Algorithm Formulas for IT2-NFS

Derivatives used in Equations (19)-21 are calculated as follows:

\[
\frac{\partial E}{\partial \sigma_{k,m}} = \sum_{p=1}^{N} \sum_{i=1}^{6} \sum_{r=1}^{R} \frac{\partial E}{\partial u_{i,p}} \frac{\partial u_{i,p}}{\partial f_{p,r}} \frac{\partial f_{p,r}}{\partial f_{p,r}} \frac{\partial f_{p,r}}{\partial \sigma_{k,m}},
\]

(B.1)

\[
\frac{\partial E}{\partial \sigma_{k,m}} = \sum_{p=1}^{N} \sum_{i=1}^{6} \sum_{r=1}^{R} \frac{\partial E}{\partial u_{i,p}} \frac{\partial u_{i,p}}{\partial f_{p,r}} \frac{\partial f_{p,r}}{\partial f_{p,r}} \frac{\partial f_{p,r}}{\partial \sigma_{k,m}},
\]

(B.2)

\[
\frac{\partial E}{\partial \sigma_{k,m}} = \sum_{p=1}^{N} \sum_{i=1}^{6} \sum_{r=1}^{R} \frac{\partial E}{\partial u_{i,p}} \frac{\partial u_{i,p}}{\partial f_{p,r}} \frac{\partial f_{p,r}}{\partial f_{p,r}} \frac{\partial f_{p,r}}{\partial \sigma_{k,m}},
\]

(B.3)

\[
\frac{\partial E}{\partial u_{i,p}} = (u_{i,p} - u_{d,i,p}),
\]

(B.4)

According to Equations (10)-18 derivative of each output with respect to the left and right limit of firing strength of each rule is calculated as follows:
\[ \frac{\partial u_{i,p}}{\partial f_{p,r}} = q \frac{z_{p,r} - u_{p,r}}{\sum_{i=1}^{N} f_{i,t}}. \]  
(B.5)

\[ \frac{\partial u_{i,p}}{\partial f_{p,r}} = (1 - q) \frac{z_{p,r} - u_{p,r}}{\sum_{i=1}^{N} f_{i,t}}. \]  
(B.6)

According to Equations (14) and (15) we have:

\[ \frac{\partial f_{p,r}}{\partial \mu_{p,k,m}} = \prod_{i=1 \neq k}^{9} \mu_{p,i,m}^{(B.7)} \]
\[ \frac{\partial f_{p,r}}{\partial \sigma_{k,m}} = \prod_{i=1 \neq k}^{9} \sigma_{k,m}^{(B.8)} \]

Finally, using Equations (4) and (5) derivatives of upper and lower membership degree of kth input in rule r, for pth input, are as following:

\[ \frac{\partial \mu_{p,k,m}}{\partial c_{1,k,m}} = \begin{cases} 0, & x_{p,k} < \frac{(c_{1,k,m} + c_{2,k,m})}{2} \\ \frac{x_{p,k} - c_{1,k,m}}{\sigma_{k,m}} \times \cdots \times \exp\left(-\frac{1}{2} \left(\frac{x_{p,k} - c_{1,k,m}}{\sigma_{k,m}}\right)^2\right), & \text{otherwise} \end{cases} \]  
(B.9)

\[ \frac{\partial \mu_{p,k,m}}{\partial c_{2,k,m}} = \begin{cases} 0, & \text{otherwise} \\ \frac{x_{p,k} - c_{2,k,m}}{\sigma_{k,m}} \times \cdots \times \exp\left(-\frac{1}{2} \left(\frac{x_{p,k} - c_{2,k,m}}{\sigma_{k,m}}\right)^2\right), & x_{p,k} < \frac{(c_{1,k,m} + c_{2,k,m})}{2} \end{cases} \]  
(B.10)

\[ \frac{\partial \sigma_{k,m}}{\partial c_{1,k,m}} = \begin{cases} 0, & \text{otherwise} \\ \frac{x_{p,k} - c_{1,k,m}}{\sigma_{k,m}} \times \cdots \times \exp\left(-\frac{1}{2} \left(\frac{x_{p,k} - c_{1,k,m}}{\sigma_{k,m}}\right)^2\right), & x_{p,k} < \frac{(c_{1,k,m} + c_{2,k,m})}{2} \end{cases} \]  
(B.11)

\[ \frac{\partial \sigma_{k,m}}{\partial c_{2,k,m}} = \begin{cases} 0, & \text{otherwise} \\ \frac{x_{p,k} - c_{2,k,m}}{\sigma_{k,m}} \times \cdots \times \exp\left(-\frac{1}{2} \left(\frac{x_{p,k} - c_{2,k,m}}{\sigma_{k,m}}\right)^2\right), & x_{p,k} < \frac{(c_{1,k,m} + c_{2,k,m})}{2} \end{cases} \]  
(B.12)

Mehran Safayani received his B.S. degree in computer engineering from University of Isfahan, Iran in 2002. Then, he received the M.Sc. and Ph.D. degrees from Sharif University of Technology, Tehran, Iran in computer architecture and artificial intelligence in 2006 and 2011 respectively. Since 2012, he is an assistant professor of Electrical and Computer Engineering at Isfahan University of Technology. His research interests include statistical pattern recognition, machine learning, soft computing and probabilistic graphical models.

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