

Control of Flexible Joint Manipulator via Variable Structure Rule-Based Fuzzy Control and Chaos Anti-Control with Experimental Validation

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Abstract :

This paper presents a variable structure rule-based fuzzy control for trajectory tracking and vibration control of a flexible joint manipulator by using chaotic anti-control. Based on Lyapunov stability theory for variable structure control and fuzzy rules, the nonlinear controller and some generic sufficient conditions for global asymptotic control are attained. The fuzzy rules are directly constructed subject to a Lyapunov function obtained from variable structure surfaces such that the error dynamics of control problem satisfy stability in the Lyapunov sense. Also in this study, the anti-control is applied to reduce the deflection angle of flexible joint system. To achieve this goal, the chaos dynamic must be created in the flexible joint system. So, the flexible joint system has been synchronized to chaotic gyroscope system. In this study, control and anti-control concepts are applied to achieve the high quality performance of flexible joint system. It is tried to design a controller which is capable to satisfy the control and anti-control aims. The performances of the proposed control are examined in terms of input tracking capability, level of vibration reduction and time response specifications. Finally, the efficacy of the proposed method is validated through experimentation on QUANSER's flexible-joint manipulator.

Keywords: Anti-control; Chaos synchronization; Chaotic gyroscope; Flexible joint, Fuzzy control, Variable structure control.

1. INTRODUCTION

The trajectory tracking control of robotic manipulators with joint flexibility has received considerable attention, owing to the complexity of the problem. Many robots incorporate harmonic drives for speed reduction, and it is known that such drives introduce torsional elasticity into the joints [1]. Industrial robots generally have elastic elements in the transmission systems, which may result in the occurrence of torsional vibrations when a fast response is required. For many manipulators, joint elasticity may arise from several sources, such as elasticity in gears, belts, tendons,

bearings, hydraulic lines, etc., and may limit the speed and dynamic accuracy achievable by control algorithms designed assuming perfect rigidity at joints. A proper choice of mathematical model for a control system design is a crucial stage in the development of control strategies for any system. This is particularly true for robotic manipulators due to their complicated dynamics [2].

Experimental evidence suggests that joint flexibility should be taken into account in both modeling and control of manipulators if high performance is to be achieved. To model this elastic behavior in the joints, the link is considered as connected to rotor through a torsional spring of stiffness K . The introduction of joint flexibility in the robot model considerably complicates the equations of

¹ Submission date: 30 , 10 , 2012

Acceptance date: 07 , 05 , 2013

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motion. In particular, the order of the related dynamics becomes twice that of the rigid robots, and the number of degrees of freedom is larger than the number of inputs, making the control task difficult.

Research on the dynamic modeling and control of flexible robots has received increased attention in the last decades. A first step towards designing an efficient control strategy for manipulators with flexible joints must be aimed at developing dynamic models that can characterize the flexibility of the joints accurately. The controller design that minimizes the effects of the flexible displacements in lightweight robots is highly demanded in many industrial and space applications that require accurate trajectory control. In control applications of robot manipulators with flexible arms are targeted either to reach a target position or to follow a prescribed trajectory. In the first case to reach a target position, a short settling time is desired while a large robot arm displacement is planned in the second case to follow a prescribed trajectory. In both cases, strong control actions are applied to the robot arm and as a result, undesired behaviors could appear if vibrations induced in the robot arm are not considered [3].

The control task of the flexible joint system is to design the appropriate control system such that the flexible joint can reach the desired position or track a prescribed trajectory precisely with minimum vibrations of the joint. Hence, to attain these objectives, various methods have been proposed which can be categorized as follows;

- Linear quadratic regulation (LQR) control [4],
- Adaptive output-feedback controller based on a back stepping design [5-7],
- Nonlinear control based on feedback linearization technique and the integral manifold technique [8,9],
- Robust control based on PD control [10], and robust H_∞ control [11],
- Fuzzy control, PD fuzzy and Neural network [12-16]
- Optimal control [17, 18],
- and so on [2, 19-24].

In this paper, a variable structure rule based fuzzy control is designed to control flexible joint

system. Based on Lyapunov stability theory for variable structure control and fuzzy rules, the nonlinear controller and some generic sufficient conditions for global asymptotic control are attained. The fuzzy rules are directly constructed subject to a Lyapunov function obtained from variable structure surfaces such that the error dynamics of control problem satisfy stability in the Lyapunov sense. Also, anti-control is applied in this study to reduce the deflection angle of flexible joint system. It means that the chaotic dynamic can be useful to control flexible joint system as an anti- control (More details are presented in section IV). In this study, to create the chaos dynamic in the flexible joint system, this system can be synchronized to the chaotic gyroscope system. Therefore, control and anti-control are applied to achieve the high quality performance of flexible joint system. It is tried to design a controller which is capable to satisfy the control and anti- control aims. The designed controller has been implemented on the QUANSER flexible joint system. Advantages of the proposed approach to control flexible joint which are not considered in previous studies are as follows: (1) while the system is synchronized with a chaotic system, the global stability of the system is guaranteed. (2) The proposed approach is capable to reduce the deflection angle of the flexible joint with the specific scale. (3) The proposed control method is robust due to be sliding surface and sliding mode control in the proposed control strategy.

This paper is organized as follows: The flexible joint manipulator and modelling of this system are described in section 2. Control problem formulation is presented in Section 3. In section 4, chaos in flexible joint and chaos synchronization are explained. Also, chaotic gyroscope system is described in this section. In section 5, the variable structure rule-based fuzzy control is designed. Finally, the implementation and results obtained from QUANSER flexible joint system are presented to show the effectiveness of proposed control method in section 6. At the end, the paper is concluded in section 7.

2. DESCRIPTION OF THE FLEXIBLE JOINT MANIPULATOR

The flexible joint manipulator system considered in this work is shown in Fig. 1, where

θ is the tip angular position and α is the deflection angle of the flexible joint. The base of the flexible joint manipulator which determines the tip angular position of the flexible joint is driven by servomotor, while the flexible joint will response based on base movement. The deflection of the joint will be determined by the flexibility of the spring as their intrinsic physical characteristics [25].

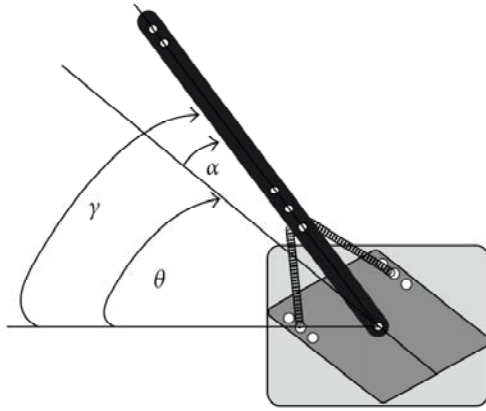


Fig.1. Flexible Joint Manipulator System.

2.1 MODELING OF THE SYSTEM

This section provides a brief description on the modeling of the flexible joint manipulator system, as a basis of a simulation environment for development and assessment of the nonlinear control. The Euler-Lagrange formulation is considered in characterizing the dynamic behavior of the system. Considering the motion of the flexible joint system on a two-dimensional plane, the potential energy of the spring can be formulated as [25]:

$$V = \frac{1}{2} K_{stiff} \alpha^2 \quad (1)$$

where K_{stiff} is the joint stiffness. The kinetic energies in the system arise from the moving hub and flexible joint can be formulated as:

$$T = \frac{1}{2} J_{eq} \dot{\theta}^2 + \frac{1}{2} J_{Arm} (\dot{\theta} + \dot{\alpha})^2 \quad (2)$$

Where J_{eq} and J_{Arm} are the equivalent inertia and total joint inertia, respectively. To obtain a closed-form dynamic model of the flexible joint, the energy expressions in Eq. (1) and Eq. (2) are applied to formulate the Lagrangian, that is:

$$L = T - V \quad (3)$$

$$L = \frac{1}{2} J_{eq} \dot{\theta}^2 + \frac{1}{2} J_{Arm} (\dot{\theta} + \dot{\alpha})^2 - \frac{1}{2} K_{stiff} \alpha^2$$

Two generalized co-ordinates are θ and α . Let

the generalized torque corresponding to the generalized tip angle be $T_{output} - B_{eq} \dot{\theta}$. Using Lagrangian's equation as follow:

$$\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = T_{output} - B_{eq} \dot{\theta} \quad (4)$$

$$\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{\alpha}} \right) - \frac{\partial L}{\partial \alpha} = 0 \quad (5)$$

The equation of motion is obtained as below:

$$J_{eq} \ddot{\theta} + J_{Arm} (\ddot{\theta} + \ddot{\alpha}) = T_{output} - B_{eq} \dot{\theta} \quad (6)$$

$$J_{Arm} (\ddot{\theta} + \ddot{\alpha}) + K_{stiff} \alpha = 0 \quad (7)$$

Where B_{eq} is the equivalent viscous damping and T_{output} is the output torque on the load from the motor, defined as:

$$T_{output} = \frac{\eta_m \eta_g K_t K_g (V_m - K_g K_m \dot{\theta})}{R_m} \quad (8)$$

Where η_m is the motor efficiency, η_g is the gearbox efficiency, K_t is the motor torque constant, K_g is the high gear ratio, K_m is the motor back-EMF constant and R_m is the armature resistance. The linear model of the uncontrolled system can be represented in a state-space form as shown in equation Eq. (9), that is:

$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \\ \dot{x}_3 = ax_2 + bx_3 + cu \\ \dot{x}_4 = dx_2 + fx_3 - cu \end{cases} \quad (9)$$

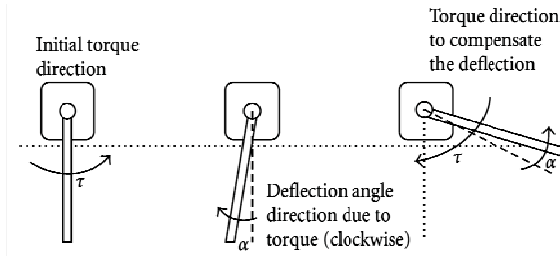
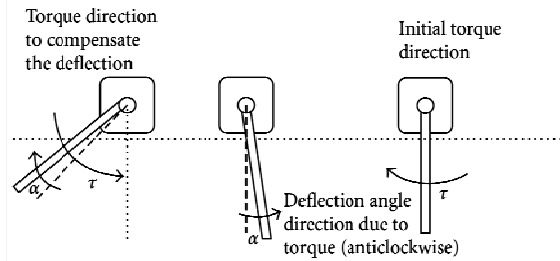
Where $x = [\theta \ \alpha \ \dot{\theta} \ \dot{\alpha}]^T$, and a, b, c, d and f are given as:

$$\begin{cases} a = \frac{K_{stiff}}{J_{eq}} \\ b = \frac{-\eta_m \eta_g K_t K_m K_g^2 + B_{eq} R_m}{J_{eq} R_m} \\ c = \frac{\eta_m \eta_g K_t K_g}{J_{eq} R_m} \\ d = -\frac{K_{stiff} (J_{eq} + J_{Arm})}{J_{eq} J_{Arm}} \\ f = \frac{\eta_m \eta_g K_t K_m K_g^2 + B_{eq} R_m}{J_{eq} R_m} \end{cases} \quad (10)$$

In Eq. (9), the input u is the input voltage of the servomotor, V_m which determines the flexible joint manipulator base movement. In this study, the values of the parameters are defined as Table.1 [12, 13, 15, 25]. Directions of torque to reduce the deflection angle when link moves anti-clockwise and clockwise are shown in Fig.2 (a) and Fig. 2 (b), respectively.

Table.1: System Parameters

Symbol	Quantity	value
R_m	Armature Resistance (Ω)	2.6
K_m	Motor Back-EMF Constant ($v.rad/s$)	0.00767
K_t	Motor Torque Constant ($N.m/s$)	0.00767
J_{Arm}	Total Arm Inertia ($kg.m^2$)	0.0035
J_{eq}	Equivalent Inertia ($kg.m^2$)	0.0026
K_g	High Gear Ratio	14:5
K_{stiff}	Joint Stiffness	1.2485
B_{eq}	Equivalent Viscous Damping ($N.m.s/rad$)	0.004
μ_g	Gearbox Efficiency	0.9
μ_m	Motor Efficiency	0.69

**Fig.2 (a): Direction of torque to reduce deflection angle when link moves anti-clockwise.****Fig.2 (b): Direction of torque to reduce deflection angle when link moves clockwise.**

3. CONTROL PROBLEM FORMULATION

In the previous section, it has been shown that the flexible joint system can be considered as an under-actuated system. In order to improve the performance of the dynamic system, we need to control the flexible joint system with a suitable motion which is beneficial for working with a particular condition. It is thus of great practical importance to develop suitable control methods. For this purpose, in this section, control problem of the flexible joint system is formulated. The

control aims are that:

- (1) Tracking the desired tip angular position,
- (2) Minimizing the deflection angle.

Therefore, the control problem is to drive the system to track a four-dimensional desired vector $X_d(t)$ as follows:

$$X_d(t) = [x_{d,1}, x_{d,2}, x_{d,3}, x_{d,4}]^T \quad (11)$$

$$= [x_{d,1}, x_{d,2}, \dot{x}_{d,1}, \dot{x}_{d,2}]^T$$

Let us define the tracking error as:

$$\begin{cases} e_c = x_1 - x_{d,1} \\ e_s = x_2 - x_{d,2} \end{cases} \quad (12)$$

Then, the error dynamics can be obtained from Eq. (12) and Eq. (9) as follow:

$$\begin{cases} \dot{e}_c = \dot{x}_1 - \dot{x}_{d,1} = x_3 - \dot{x}_{d,1} \\ \dot{e}_s = \dot{x}_2 - \dot{x}_{d,2} = x_4 - \dot{x}_{d,2} \end{cases} \quad (13)$$

To achieve the control input, it is necessary to differentiate Eq. (13). From Eq. (13) and Eq. (9), we obtain:

$$\begin{cases} \ddot{e}_c = \ddot{x}_3 - \ddot{x}_{d,1} = ax_2 + bx_3 - \ddot{x}_{d,1} + cu \\ \quad = g_1(x_2, x_3, x_{d,1}) + cu \\ \ddot{e}_s = \ddot{x}_4 - \ddot{x}_{d,2} = dx_2 + fx_3 - \ddot{x}_{d,2} - cu \\ \quad = g_2(x_2, x_3, x_{d,2}) - cu \end{cases} \quad (14)$$

Therefore, the control problem can be formulated as follow:

$$\begin{cases} \dot{e}_{c1} = e_{c2} \\ \dot{e}_{c2} = g_1(x_2, x_3, x_{d,1}) + cu \\ \dot{e}_{s1} = e_{s2} \\ \dot{e}_{s2} = g_2(x_2, x_3, x_{d,2}) - cu \end{cases} \quad (15)$$

Define the error vector as:

$$E(t) = [e_{c1}(t), e_{c2}(t), e_{s1}(t), e_{s2}(t)]^T \quad (16)$$

The control goal considered in this section is that for any given target orbit $X_d(t)$, the controller is designed such that the resulting tracking error vector satisfies:

$$\lim_{t \rightarrow \infty} \|E(t)\| \rightarrow 0 \quad (17)$$

where $\|\cdot\|$ is the Euclidean norm of a vector.

4. CHAOS IN FLEXIBLE JOINT

In the previous section, the control aims have been explained. In this study, the desired vector $X_d(t)$ will be considered as follow: (1) $x_{d,1}$ is the trajectory of the reference input such as pulse

and sinusoid trajectories shown in result section. (2) $x_{d,2}$ is considered as a proportional trajectory of a chaotic mechanical system such as a chaotic gyroscope system. This choice has been proposed to use the properties of chaotic systems described in the next section.

4.1 CHAOS SYNCHRONIZATION TO CONTROL OF FLEXIBLE JOINT

Dynamic chaos is a very interesting nonlinear effect which has been intensively studied during the last three decades. Chaos control can be mainly divided into two categories [26]: one is the suppression of the chaotic dynamical behavior and the other is to generate or enhance chaos in a nonlinear system.

In this study, chaos synchronization [27] has been suggested to create the chaotic behavior in the flexible joint system. Basically, the chaos synchronization problem means making two systems oscillate in a synchronized manner. Given a chaotic system considered as the master system, and another system considered as the slave system, the dynamical behaviors of these two systems may be identical after a transient time when the slave system is driven by a control input. Different types of synchronization have been found in interacting chaotic systems, such as generalized projective synchronization that the master and slave vectors synchronize up to a constant scaling factor α (a proportional relation) [28-30].

In this study, the flexible joint system and the chaotic gyroscope system have been considered as the slave and the master systems, respectively.

Notice that, the second state of the flexible joint will be synchronized to the first state of the chaotic gyroscope system. This synchronization is the generalized projective synchronization with a constant scaling factor α . So,

$$x_{d,2} = \alpha x_{1gyro} \quad (18)$$

where α is a very small scalar value in the range of 10^{-n} , $3 < n < 5$.

In this section, chaos synchronization is used as the anti-control to control the flexible joint system. In the next part, dynamics of the chaotic gyroscope system are described.

4.2 CHAOTIC GYROSCOPE SYSTEM

The symmetric gyroscope mounted on a vibrating base is shown in Fig. 3. The dynamics of a symmetrical gyro with linear-plus-cubic damping of angle θ can be expressed as [31]:

$$\ddot{\theta} + \alpha_1^2 \frac{(1 - \cos \theta)^2}{\sin^3 \theta} - \beta_1 \sin \theta + c_1 \dot{\theta} + c_2 \dot{\theta}^3 = f \sin \omega t \sin \theta \quad (19)$$

where $f \sin \omega t$ is a parametric excitation, $c_1 \dot{\theta}$ and $c_2 \dot{\theta}^3$ are linear and nonlinear damping terms, respectively and $\alpha_1^2 (1 - \cos \theta)^2 / \sin^3 \theta - \beta_1 \sin \theta$ is a nonlinear resilience force. According to [31], in a symmetric gyro mounted on a vibrating base, the precession and the spin angles have cyclic motions and hence their momentum integrals are constant and equal to each other. So the governing equations of motion depend only on the mutational angle θ . Using Routh's procedure and assuming a linear-plus-cubic form for dissipative force, Eq. (19) is obtained [31]. Given the states $x_1 = \theta$ and $x_2 = \dot{\theta}$ and $g(\theta) = \alpha_1^2 (1 - \cos \theta)^2 / \sin^3 \theta - \beta_1 \sin \theta$, Eq. (19) can be rewritten as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = g(x_1) - c_1 x_1 - c_2 x_2^3 + (\beta_1 + f \sin \omega t) \sin(x_1) \end{cases} \quad (20)$$

This gyro system exhibits complex dynamics and has been studied by [31] for values of f in the range $32 < f < 36$ and constant values of $\alpha_1^2 = 100$, $\beta_1 = 1$, $c_1 = 0.5$, $c_2 = 0.5$ and $\omega = 2$. Fig. 4 illustrate the irregular motion exhibited by this system for $f = 35.5$ and initial conditions of $(x_1, x_2) = (1, -1)$. The chaotic gyroscope system, presented in Eq. (20), is a popular chaotic system that many studies have been carried out about it such as [31-33].

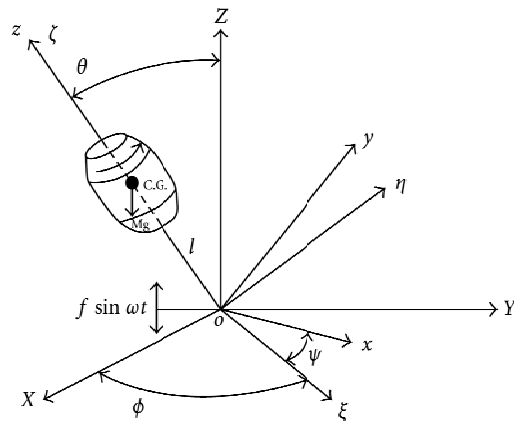


Fig. 3: A schematic diagram of a symmetric gyroscope.

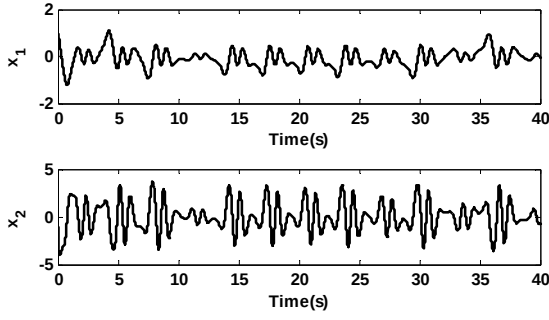


Fig.4: Time series of x_1 and x_2 .

In the next section, the control input will be designed via variable structure rule-based fuzzy control to achieve the control objective presented in previous section.

5. VARIABLE STRUCTURE RULE-BASED FUZZY CONTROL

In this section, intelligent nonlinear control is designed. First, switching surfaces are defined. Then, the variable structure rule-based fuzzy control is designed. Achieving the fuzzy rules is completely explained.

5.1 SWITCHING SURFACE

Using the VSC control method to control the flexible joint system, involves two basic steps; (1) Selecting an appropriate sliding surface such that the sliding motion on the sliding manifold is stable and ensures $\lim_{t \rightarrow \infty} \|E(t)\| \rightarrow 0$. (2) Establishing a robust control law which guarantees the existence of the sliding manifold $S(t) = 0$.

The sliding surfaces are defined as [34]:

$$S(t) = \left(\frac{d}{dt} + \delta \right)^{n-1} e(t) \quad (21)$$

where $S(t) \in R$ and δ is a real positive constant parameter. Differentiating Eq. (21) with respect to time as follow:

$$\dot{S}(t) = \left(\frac{d}{dt} + \delta \right)^n e(t) \quad (22)$$

The rate of convergence of the sliding surface is governed by the value assigned to parameter δ . In this study, define two sliding surfaces as:

$$S_1(t) = e_{c2}(t) + \delta_1 e_{c1}(t) \quad (23)$$

$$S_2(t) = e_{s2}(t) + \delta_2 e_{s1}(t) \quad (24)$$

Differentiating Eq. (23) and Eq. (24) with respect to time as follow:

$$\dot{S}_1(t) = \dot{e}_{c2}(t) + \delta_1 \dot{e}_{c1}(t) \quad (25)$$

$$\dot{S}_2(t) = \dot{e}_{s2}(t) + \delta_2 \dot{e}_{s1}(t) \quad (26)$$

Substituting Eq. (15) into Eq. (25) and Eq. (26), then:

$$\dot{S}_1(t) = g_1(\cdot) + \delta_1 e_{c2} + cu \quad (27)$$

$$\dot{S}_2(t) = g_2(\cdot) + \delta_2 e_{s2} - cu \quad (28)$$

5.2 VARIABLE STRUCTURE RULE-BASED FUZZY CONTROL

A set of the fuzzy linguistic rules based on expert knowledge are applied to design the control law of fuzzy logic control. To overcome the trail-and-error tuning of the membership functions and rule base, the fuzzy rules are directly defined such that the error dynamics satisfies stability in the Lyapunov sense. The basic fuzzy logic system is composed of five function blocks [35]: (1) A rule base contains a number of fuzzy if-then rules. (2) A database defines the membership functions of the fuzzy sets used in the fuzzy rules. (3) A decision-making unit performs the inference operations on the rules. (4) A fuzzification interface transforms the crisp inputs into degrees of match with linguistic value. (5) A defuzzification interface transforms the fuzzy results of the inference into a crisp output.

The fuzzy rule base consists of a collection of fuzzy if-then rules expressed as the form if a is A then b is B, where a and b denote linguistic variables, A and B represent linguistic values that are characterized by membership functions. All of the fuzzy rules can be used to construct the fuzzy associated memory.

In this study, the two FLC are designed as; the signal (S_1, S_2) in Eq. (15) as the antecedent part of the proposed FLC to design the control input u that will be used in the consequent part of the proposed FLC.

$$u = FLC(S_1, S_2) \quad (29)$$

where the FLC accomplishes the objective to stabilize the error dynamics Eq. (15). The i th if-then rule of the fuzzy rule base of the FLC is of

the following form:

Rule i: if S_1 is X_1 and S_2 is X_2 then

$$u_{Li} \equiv f_i(S_1, S_2) \quad (30)$$

where X_1 and X_2 are the input fuzzy sets, u_{Li} is the output which is the analytical function $f_i(\cdot)$ of the input variables (S_1, S_2).

For given input values of the process variables, their degrees of membership μ_{xi} , $i = 1, 2, \dots, n$ called rule-antecedent weights are calculated. The centroid defuzzifier evaluates the output of all rules as follows:

$$u = \frac{\sum_{i=1, \mu \neq 0}^n \mu_i \cdot u_{Li}}{\sum_{i=1, \mu \neq 0}^n \mu_i} \quad (31)$$

$$\mu_i = \mu_{x1}(e_{c2})\mu_{x2}(e_{s2})$$

Table 2 lists the fuzzy rule base in which the input variables in the antecedent part of the rules are S_1 and S_2 , and the output variable in the consequent is u_{Li} .

Using P, Z and N as input fuzzy sets representing 'positive', 'zero' and 'negative', respectively. The Gaussian membership function is considered. The combination of the two input variables (S_1, S_2) forms $n = 9$ heuristic rules in Table1 and each rules belongs to one of the three fuzzy sets P, Z and N. The rules in Table2 are read as, taking rule 1 in Table2 as an example, 'rule 1: if input1 S_1 is P and input2 S_2 is P, then output is u_{L1} '.

To solve the control problem presented in Eq. (15), define a Lyapunov function as follow:

$$V(t) = \frac{1}{2} S_1^2(t) + \frac{1}{2} S_2^2(t) \quad (32)$$

Differentiating Eq. (32) with respect to time as:

$$\dot{V} = S_1 \dot{S}_1 + S_2 \dot{S}_2 \quad (33)$$

Substituting Eq. (27) and Eq. (28) into Eq. (33), then:

$$\dot{V} = S_1[g_1(\cdot) + \delta_1 e_{c2} + cu] + S_2[g_2(\cdot) + \delta_2 e_{s2} - cu] = S_1 A + S_2 B \quad (34)$$

The corresponding requirement of Lyapunov stability is [34]:

$$\dot{V} < 0 \quad (35)$$

If $(S_1.A < 0)$ and $(S_2.B < 0)$, then the Lyapunov stability will be satisfied. The following cases will satisfy all the stability

conditions. This discussion is separately presented for A and B terms.

Table.2: Rule-Table of FLC

Rule No.	Antecedent		Consequent
	S_1	S_2	u_{Li}
1	P	P	$u_{L1} = -\frac{1}{2}[e_{s1} + g_2(\cdot) + e_{c1} + g_1(\cdot)]$
2	P	Z	$u_{L2} = \frac{1}{2c}[\delta_2 e_{s2} + g_2(\cdot) + \eta \operatorname{sgn}(S_2) - \delta_1 e_{c2} - g_1(\cdot)] - \lambda$
3	P	N	$u_{L3} = \frac{1}{2c}[\delta_2 e_{s2} + g_2(\cdot) - \delta_1 e_{c2} - g_1(\cdot)] - \lambda$
4	Z	P	$u_{L4} = \frac{1}{2c}[\delta_2 e_{s2} + g_2(\cdot) - \eta \operatorname{sgn}(e_{s2}) - \delta_1 e_{c2} - g_1(\cdot)] + \lambda$
5	Z	Z	$u_{L5} = 0$
6	Z	N	$u_{L6} = \frac{1}{2c}[\delta_2 e_{s2} + g_2(\cdot) - \eta \operatorname{sgn}(S_1) - \delta_1 e_{c2} - g_1(\cdot)] - \lambda$
7	N	P	$u_{L7} = \frac{1}{2c}[\delta_2 e_{s2} + g_2(\cdot) - \delta_1 e_{c2} - g_1(\cdot)] + \lambda$
8	N	Z	$u_{L8} = \frac{1}{2c}[\delta_2 e_{s2} + g_2(\cdot) + \eta \operatorname{sgn}(S_2) - \delta_1 e_{c2} - g_1(\cdot)] + \lambda$
9	N	N	$u_{L9} = \frac{1}{2c}[\delta_2 e_{s2} + g_2(\cdot) + \delta_1 e_{c2} + g_1(\cdot)]$

Consequents of the rules presented in Table.2 are obtained as follows:

Rule.1:

If $S_1 > 0$ and $S_2 > 0$ then,

$$\begin{aligned} A &= g_1(\cdot) + \delta_1 e_{c2} + cu < 0 \\ B &= g_2(\cdot) + \delta_2 e_{s2} - cu < 0 \end{aligned} \quad (36)$$

Considering the consequent part of Rule.1:

$$g_1(\cdot) + \delta_1 e_{c2} + cu < 0 \quad (37)$$

$$g_2(\cdot) + \delta_2 e_{s2} - cu < 0 \quad (38)$$

Eq. (37) and Eq. (38) can be simplified:

$$g_2(\cdot) + \delta_2 e_{s2} < cu < -(g_1(\cdot) + \delta_1 e_{c2}) \quad (39)$$

Let us choose the control input as follow such that the Eq. (39) is satisfied:

$$u_{L1} = -\frac{1}{2}[e_{s1} + g_2(\cdot) + e_{c1} + g_1(\cdot)] \quad (40)$$

Rule.2:

If $S_1 > 0$ and $S_2 \in \text{zero}$ then,

$$\delta_1 e_{c2} + g_1(\cdot) + cu < 0 \quad (41)$$

$$\delta_2 e_{s2} + g_2(\cdot) - cu = -\eta \operatorname{sgn}(e_{s2}) \quad (42)$$

where η is a positive constant value.

Eq. (41) and Eq. (42) can be simplified:

$$2cu < [\delta_2 e_{s2} + g_2(\cdot) + \eta \operatorname{sgn}(S_2) - \delta_1 e_{c2} - g_1(\cdot)] \quad (43)$$

Let us choose the control input as follow such that the Eq. (43) is satisfied:

$$u_{L2} = \frac{1}{2c} [\delta_2 e_{s2} + g_2(\cdot) + \eta \operatorname{sgn}(S_2) - \delta_1 e_{c2} - g_1(\cdot)] - \lambda \quad (44)$$

where λ is a positive constant value.

Rule.3:

If $S_1 > 0$ and $S_2 < 0$ then

$$\begin{aligned} A &= g_1(\cdot) + \delta_1 e_{c2} + cu < 0 \\ B &= g_2(\cdot) + \delta_2 e_{s2} - cu > 0 \end{aligned} \quad (45)$$

Considering the consequent part of Rule.3:

$$\delta_1 e_{c2} + g_1(\cdot) + cu < 0 \quad (46)$$

$$\delta_2 e_{s2} + g_2(\cdot) - cu > 0 \quad (47)$$

Eq. (46) and Eq. (47) can be simplified:

$$2cu < -g_1(\cdot) - \delta_1 e_{c2} + g_2(\cdot) + \delta_2 e_{s2} \quad (48)$$

Let us choose the control input as follow such that the Eq. (48) is satisfied:

$$u_{L3} = \frac{1}{2c} [\delta_2 e_{s2} + g_2(\cdot) - \delta_1 e_{c2} - g_1(\cdot)] - \lambda \quad (49)$$

where λ is a positive constant value.

Rule.4:

If $S_1 \in \text{zero}$ and $S_2 > 0$ then,

$$\delta_2 e_{s2} + g_2(\cdot) - cu < 0 \quad (50)$$

$$\delta_1 e_{c2} + g_1(\cdot) + cu = -\eta \operatorname{sgn}(S_1) \quad (51)$$

where η is a positive constant value.

Eq. (50) and Eq. (51) can be simplified:

$$2cu > [\delta_2 e_{s2} + g_2(\cdot) - \eta \operatorname{sgn}(e_{c2}) - \delta_1 e_{c2} - g_1(\cdot)] \quad (52)$$

Let us choose the control input as follow such that the Eq. (52) is satisfied:

$$u_{L4} = \frac{1}{2c} [\delta_2 e_{s2} + g_2(\cdot) - \eta \operatorname{sgn}(e_{s2}) - \delta_1 e_{c2} - g_1(\cdot)] + \lambda \quad (53)$$

where λ is a positive constant value.

Rule.5:

For rule 5 in Table 2, the error states e_{c2} and e_{s2} are zeros. This condition is included in the other rules, and we define $u_{L5} = 0$.

Rule.6:

If $S_1 \in \text{zero}$ and $S_2 < 0$ then,

$$\delta_2 e_{s2} + g_2(\cdot) - cu > 0 \quad (54)$$

$$\delta_1 e_{c2} + g_1(\cdot) + cu = -\eta \operatorname{sgn}(S_1) \quad (55)$$

where η is a positive constant value.

Eq. (54) and Eq. (55) can be simplified:

$$2cu < [\delta_2 e_{s2} + g_2(\cdot) - \mu \operatorname{sgn}(e_{c2}) - \delta_1 e_{c2} - g_1(\cdot)] \quad (56)$$

Let us choose the control input as follow such that the Eq. (56) is satisfied:

$$u_{L6} = \frac{1}{2c} [\delta_2 e_{s2} + g_2(\cdot) - \eta \operatorname{sgn}(S_1) - \delta_1 e_{c2} - g_1(\cdot)] - \lambda \quad (57)$$

where λ is a positive constant value.

Rule.7:

If $S_1 < 0$ and $S_2 > 0$ then,

$$A = g_1(\cdot) + \delta_1 e_{c2} + cu > 0 \quad (58)$$

$$B = g_2(\cdot) + \delta_2 e_{s2} - cu < 0$$

Considering the consequent part of Rule.7:

$$\delta_1 e_{c2} + g_1(\cdot) + cu > 0 \quad (59)$$

$$\delta_2 e_{s2} + g_2(\cdot) - cu < 0 \quad (60)$$

Eq. (59) and Eq. (60) can be simplified:

$$2cu > -\delta_1 e_{c2} - g_1(\cdot) + g_2(\cdot) + \delta_2 e_{s2} \quad (61)$$

Let us choose the control input as follow such that the Eq. (61) is satisfied:

$$u_{L7} = \frac{1}{2c} [\delta_2 e_{s2} + g_2(\cdot) - \delta_1 e_{c2} - g_1(\cdot)] + \lambda \quad (62)$$

where λ is a positive constant value.

Rule.8:

If $S_1 < 0$ and $S_2 \in \text{zero}$ then,

$$\delta_1 e_{c2} + g_1(\cdot) + cu > 0 \quad (63)$$

$$\delta_2 e_{s2} + g_2(\cdot) - cu = -\eta \operatorname{sgn}(S_2) \quad (64)$$

where η is a positive constant value.

Eq. (63) and Eq. (64) can be simplified:

$$2cu > [\delta_2 e_{s2} + g_2(\cdot) + \eta \operatorname{sgn}(S_2) - \delta_1 e_{c2} - g_1(\cdot)] \quad (65)$$

Let us choose the control input as follow such that the Eq. (65) is satisfied:

$$u_{L8} = \frac{1}{2c} [\delta_2 e_{s2} + g_2(\cdot) + \eta \operatorname{sgn}(S_2) - \delta_1 e_{c2} - g_1(\cdot)] + \lambda \quad (66)$$

where λ is a positive constant value.

Rule.9:

If $S_1 < 0$ and $S_2 < 0$ then

$$A = g_1(\cdot) + \delta_1 e_{c2} + cu > 0 \quad (67)$$

$$B = g_2(\cdot) + \delta_2 e_{s2} - cu > 0$$

Considering the consequent part of Rule.9:

$$\delta_1 e_{c2} + g_1(\cdot) + cu > 0 \quad (68)$$

$$\delta_2 e_{s2} + g_2(\cdot) - cu > 0 \quad (69)$$

Eq. (68) and Eq. (69) can be simplified:

$$2cu < \delta_1 e_{c2} + g_1(\cdot) + \delta_2 e_{s2} + g_2(\cdot) \quad (70)$$

Let us choose the control input as follow such that the Eq. (70) is satisfied:

$$u_{L9} = \frac{1}{2c} [\delta_2 e_{s2} + g_2(\cdot) + \delta_1 e_{c2} + g_1(\cdot)] \quad (71)$$

Therefore, all of the rules in the FLC can lead to Lyapunov stable subsystems under the same Lyapunov function Eq. (34). Furthermore, the closed-loop rule-based system Eq. (15) is asymptotically stable for each derivate of the Lyapunov function that satisfies $\dot{V} < 0$ in Table 2. That is, the error states guarantee convergence to zero.

6. IMPLEMENTATION AND RESULTS

In this section, the proposed control has been implemented and tested within the simulation environment of the flexible joint manipulator and the corresponding results are presented.

Laboratory system use microprocessor of personal computer (PC) for the simulation and software development as well as for the real-time

control. Control algorithm, designed and simulated in MATLAB/Simulink environment, uses graphically oriented software interface for the real-time code generation. This application oriented code is running in the real time under the same PC as for simulation. Details about experimental test bench can be found in [25].

The generated control signal has been applied to the flexible joint manipulator through QUANSER's interfacing hardware board. The fourth order Runge-Kutta algorithm are applied to solve the sets of differential equations related to the systems with a time grid of 0.001. Also, the scaling factor is considered as $\alpha = 0.0001$.

Implementation results have been investigated as follow:

The flexible joint manipulator is required to follow a pulse-trajectory of 28.65° ($=0.5$ rad) with the frequency 0.1Hz.

Fig.5 is corresponding to a pulse-trajectory of 28.65° . As Fig. 5(a) shows the flexible joint manipulator tracks the pulse-trajectory of 28.65° . Fig 5(b) demonstrates deflection angle. It is observable that deflection angle amplitude's range is satisfactory and it has a suitable damping ratio.

The flexible joint manipulator is required to follow a sinusoid-trajectory with amplitude 28.65° ($=0.5$ rad) and with the frequency 0.1Hz.

Fig.6 is corresponding to a sinusoid-trajectory of 28.65° with the frequency 0.1Hz. As Fig. 6(a) shows the flexible joint manipulator tracks the sinusoid-trajectory of 28.65° . Fig 6(b) demonstrates deflection angle. It is observable that deflection angle amplitude's range is satisfactory and it has a suitable damping ratio.

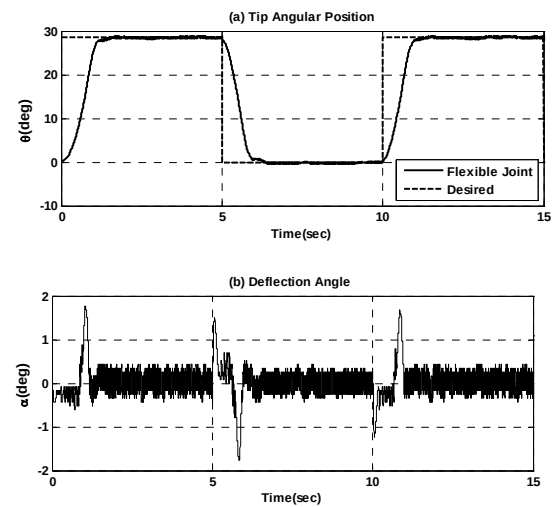


Fig.5: (Desired: pulse-trajectory of 28.65°)
(a) Tip angular position of the flexible joint manipulator, (b) Deflection angle the flexible joint manipulator.

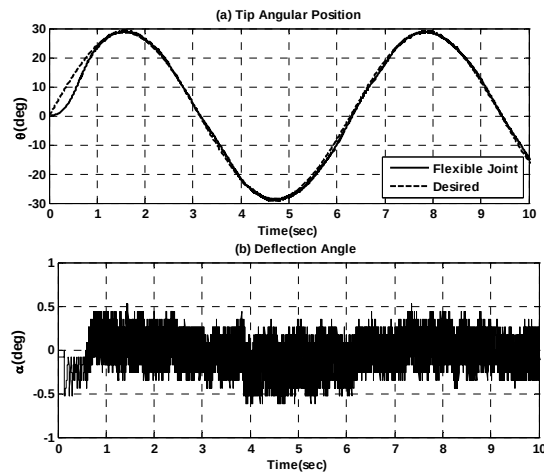


Fig.6: (Desired: sinusoid-trajectory with amplitude of 28.65°)

(a) Tip angular position of the flexible joint manipulator, (b) Deflection angle the flexible joint manipulator.

It is noted that the flexible joint manipulator reaches the required position within 1.5 sec, without overshoot. However, a noticeable amount of vibration occurs during movement of the manipulator. It is noted that the maximum deflection angle is $\pm 1.5^\circ$.

The system responses namely the tip angular position and deflection angle are observed. Time response specifications are summarized on Table.3.

Table.3: Magniyude of Specifications of Tip Angular Position

Magnitude (deg)		Desired Trajectories	
		Pulse 28.65°	Sinusoid 28.65°
Specifications of tip angular position response	Settling time (Sec)	1.5	Not Defined
	Rise time (Sec)	0.7	Not Defined
	Overshoot (Percent %)	0	Not Defined

The achieved results are illustrated that the designed controller provides higher level of vibration reduction as compared to some studies such as [13, 17, 18]. Also, the high performance of input tracking is achieved. The speed of the response is slightly improved at the expenses of decrease in the level of vibration reduction. It is concluded that the proposed controller is capable of reducing the system vibration while maintaining the input tracking performance of

the manipulator.

Therefore, the proposed control is capable of reducing the system vibration while maintaining the trajectory tracking performance of the manipulator.

7. CONCLUSION

In this paper, a variable structure rule-based fuzzy control has been designed for trajectory tracking control of flexible joint manipulator system by using chaotic anti-control. The anti-control concept is applied to reduce the deflection angle of flexible joint system. So, the chaos dynamic must be created in the flexible joint system and the flexible joint system has been synchronized to chaotic gyroscope system. Therefore, the control and anti-control concepts are applied to achieve the high quality performance of flexible joint system.

The performances of the control schemes have been evaluated in terms of input tracking capability, level of vibration reduction, and time response specifications.

Experimental results have shown that the proposed approach is effective in practice. Acceptable performance in input tracking control has been achieved with designed controller. Moreover, a significant reduction in the system vibration has been achieved with the anti-control concepts. The obtained results have been demonstrated that designed controller provide higher level of vibration reduction. Also, the high performance of input tracking has been achieved. The speed of the response is slightly improved at the expenses of decrease in the level of vibration reduction. It is concluded that the proposed controller is capable of reducing the system vibration while maintaining the input tracking performance of the manipulator.

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